Conformal defects: A bridge between local and nonlocal physics

Connor Behan

ICTP-SAIFR

2024-10-11

[1703.03430, 1703.05325] with L. Rastelli, S. Rychkov, B. Zan
[1810.07199]
[2009.03336, 2111.04747] with L. Di Pietro, E. Lauria, B. C. van Rees
[2311.02742] with E. Lauria, M. Nocchi, P. van Vliet

Motivation

Why do we like conformal field theories (CFTs)?

Motivation

Why do we like conformal field theories (CFTs)?

1. They are more symmetric than "typical" QFTs.

Translations	$x'_\mu = x_\mu + a_\mu$
Rotations	$x'_\mu = \Lambda_\mu^{\ \nu} x_\nu$
Dilations	$x'_\mu = \lambda x_\mu$
Special	$x'_\mu = \frac{x_\mu - b_\mu x^2}{1 - 2b \cdot x + b^2 x^2}$

2. They describe universal end points of RG flows.

$$H = -J \sum_{i,j} \sigma_i \sigma_j$$

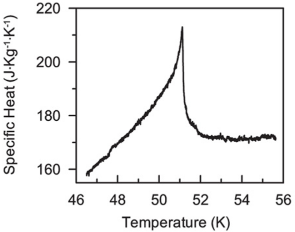
$$S = \int \frac{1}{2}(\partial\phi)^2 + \lambda\phi^4 d^d x$$

Motivation

Why do we like conformal field theories (CFTs)?

1. They are more symmetric than "typical" QFTs.

Translations	$x'_\mu = x_\mu + a_\mu$
Rotations	$x'_\mu = \Lambda_\mu{}^\nu x_\nu$
Dilations	$x'_\mu = \lambda x_\mu$
Special	$x'_\mu = \frac{x_\mu - b_\mu x^2}{1 - 2b\cdot x + b^2 x^2}$

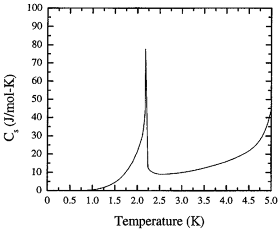


[Olega, Salazar, Bunkov; 2014]

2. They describe universal end points of RG flows.

$$H = -J \sum_{i,j} \sigma_i \sigma_j$$

$$S = \int \frac{1}{2}(\partial\phi)^2 + \lambda\phi^4 d^d x$$



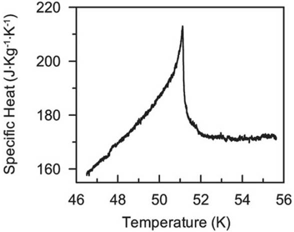
[Donnelly, Barenghi; 1998]

Motivation

Why do we like conformal field theories (CFTs)?

1. They are more symmetric than "typical" QFTs.

Translations	$x'_\mu = x_\mu + a_\mu$
Rotations	$x'_\mu = \Lambda_\mu{}^\nu x_\nu$
Dilations	$x'_\mu = \lambda x_\mu$
Special	$x'_\mu = \frac{x_\mu - b_\mu x^2}{1 - 2b\cdot x + b^2 x^2}$

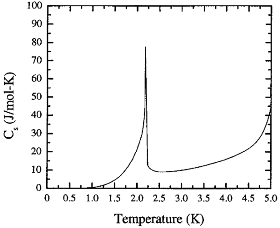


[Olega, Salazar, Bunkov; 2014]

2. They describe universal end points of RG flows.

$$H = -J \sum_{i,j} \sigma_i \sigma_j$$

$$S = \int \frac{1}{2} (\partial\phi)^2 + \lambda \phi^4 d^d x$$



[Donnelly, Barenghi; 1998]

What should we do with CFTs... bootstrap them!

Motivation

How do we bootstrap a CFT?

Motivation

How do we bootstrap a CFT?

1. Focus on the set of local operators $\mathcal{O}_\Delta^{\mu_1 \dots \mu_\ell}(x)$.

2. Demand consistency of their correlation functions on $\mathbb{R}^d$.

Motivation

How do we bootstrap a CFT?

1. Focus on the set of local operators $\mathcal{O}_\Delta^{\mu_1 \dots \mu_\ell}(x)$.

2. Demand consistency of their correlation functions on $\mathbb{R}^d$.

$$\langle \phi(x_1) \phi(x_2) \rangle = \frac{1}{|x_{12}|^{2\Delta}}$$

$$\langle \phi_1(x_1) \phi_2(x_2) \phi_3(x_3) \rangle = \frac{\lambda_{123}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3} |x_{13}|^{\Delta_1+\Delta_3-\Delta_2} |x_{23}|^{\Delta_2+\Delta_3-\Delta_1}}$$

$$\phi_1(x_1)\phi_2(x_2) = \sum_{\mathcal{O}} \frac{\lambda_{12\mathcal{O}}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta}} C_{(\mu)}(x_{12}, \partial_2) \mathcal{O}^{(\mu)}(x_2)$$

Motivation

How do we bootstrap a CFT?

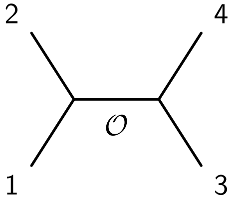
1. Focus on the set of local operators $\mathcal{O}_\Delta^{\mu_1 \dots \mu_\ell}(x)$.

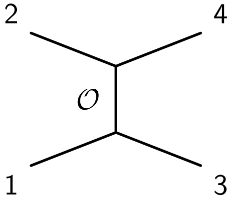
2. Demand consistency of their correlation functions on $\mathbb{R}^d$.

$$\langle \phi(x_1)\phi(x_2) \rangle = \frac{1}{|x_{12}|^{2\Delta}}$$

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3) \rangle = \frac{\lambda_{123}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}|x_{13}|^{\Delta_1+\Delta_3-\Delta_2}|x_{23}|^{\Delta_2+\Delta_3-\Delta_1}}$$

$$\phi_1(x_1)\phi_2(x_2) = \sum_{\mathcal{O}} \frac{\lambda_{12\mathcal{O}}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta}} C_{(\mu)}(x_{12}, \partial_2)\mathcal{O}^{(\mu)}(x_2)$$





[Rattazzi, Rychkov, Tonni, Vichi; 0807.0004]

[Kos, Poland, Simmons-Duffin, Vichi; 1603.04436]

$$\Delta_\sigma = 0.518149(1)$$
$$\Delta_\varepsilon = 1.412625(10)$$

Motivation

This is only a first step...

1. Focus on the set of local operators $\mathcal{O}_\Delta^{\mu_1 \dots \mu_\ell}(x)$.

2. Demand consistency of their correlation functions on $\mathbb{R}^d$.

Motivation

This is only a first step...

1. Focus on the set of local operators $\mathcal{O}_{\Delta}^{\mu_1 \dots \mu_\ell}(x)$.

2. Demand consistency of their correlation functions on $\mathbb{R}^d$.

$\hookrightarrow$ Bootstrap CFTs with defects! $\hookleftarrow$

Motivation

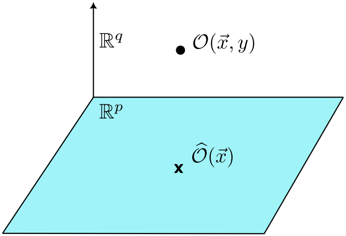
This is only a first step...

1. Focus on the set of local operators $\mathcal{O}_{\Delta}^{\mu_1 \dots \mu_\ell}(x)$.

2. Demand consistency of their correlation functions on $\mathbb{R}^d$.

↳ Bootstrap CFTs with defects! ↲

If the defect is $\mathbb{R}^p$, symmetry breaking is $SO(d+1,1) \to SO(p+1,1) \times SO(q)$ where $d = p + q$.



Motivation

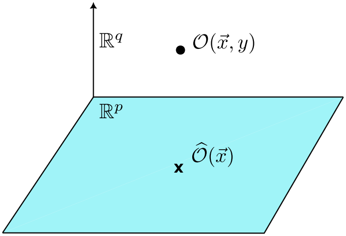
This is only a first step...

1. Focus on the set of local operators $\mathcal{O}_\Delta^{\mu_1 \dots \mu_\ell}(x)$.

2. Demand consistency of their correlation functions on $\mathbb{R}^d$.

$\hookrightarrow$ Bootstrap CFTs with defects! $\hookleftarrow$

If the defect is $\mathbb{R}^p$, symmetry breaking is $SO(d+1,1) \to SO(p+1,1) \times SO(q)$ where $d = p + q$.



Same $\left\langle \widehat{\mathcal{O}}\widehat{\mathcal{O}} \right\rangle$ and $\left\langle \widehat{\mathcal{O}}_1\widehat{\mathcal{O}}_2\widehat{\mathcal{O}}_3 \right\rangle$ but

$$\langle \phi(\vec{x}, y) \rangle = \frac{a_\phi}{|y|^\Delta}$$

$$\left\langle \phi(\vec{x}, y)\hat{\phi}(0) \right\rangle = \frac{b_{\phi\hat{\phi}}}{|y|^{\Delta - \widehat{\Delta}}(|\vec{x}|^2 + y^2)^{\widehat{\Delta}}}$$

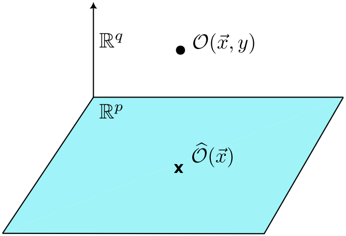
This is only a first step...

1. Focus on the set of local operators $\mathcal{O}_{\Delta}^{\mu_1 \dots \mu_\ell}(x)$.

2. Demand consistency of their correlation functions on $\mathbb{R}^d$.

↳ Bootstrap CFTs with defects! ↲

If the defect is $\mathbb{R}^p$, symmetry breaking is $SO(d+1,1) \to SO(p+1,1) \times SO(q)$ where $d = p + q$.



Same $\left\langle \widehat{\mathcal{O}}\widehat{\mathcal{O}} \right\rangle$ and $\left\langle \widehat{\mathcal{O}}_1\widehat{\mathcal{O}}_2\widehat{\mathcal{O}}_3 \right\rangle$ but

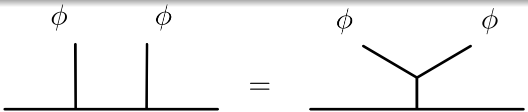
$$\langle \phi(\vec{x}, y) \rangle = \frac{a_\phi}{|y|^\Delta}$$

$$\left\langle \phi(\vec{x}, y)\hat{\phi}(0) \right\rangle = \frac{b_{\phi\hat{\phi}}}{|y|^{\Delta - \widehat{\Delta}}(|\vec{x}|^2 + y^2)^{\widehat{\Delta}}}$$

Consider bulk-defect OPE:

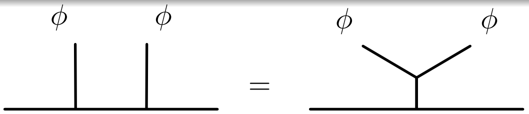
$$\phi(\vec{x}, y) = \sum_{\widehat{\mathcal{O}}} \frac{b_{\phi\widehat{\mathcal{O}}}}{|y|^{\Delta - \widehat{\Delta}}} B(y, \partial_x)\widehat{\mathcal{O}}(\vec{x})$$

The defect bootstrap



[Liendo, Rastelli, van Rees; 1210.04258]
[Billo, Goncalves, Lauria, Meineri; 1601.02883]

The defect bootstrap



[Liendo, Rastelli, van Rees; 1210.04258]
[Billo, Goncalves, Lauria, Meineri; 1601.02883]

$$\sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} v^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(u, v) - \sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} u^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(v, u) = 0$$

$$\sum_{\widehat{\mathcal{O}}} b^2_{\phi\widehat{\mathcal{O}}} f^{(d)}_{\widehat{\mathcal{O}}}(\xi, \eta) - \sum_{\mathcal{O}} \lambda_{\phi\phi\mathcal{O}} a_{\mathcal{O}} \xi^{-\Delta_\phi} f^{(b)}_{\mathcal{O}}(\xi, \eta) = 0$$

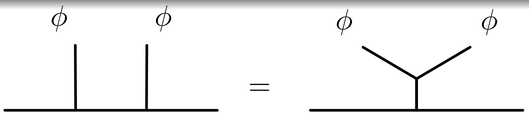
The defect bootstrap

[Liendo, Rastelli, van Rees; 1210.04258]
[Billo, Goncalves, Lauria, Meineri; 1601.02883]

$$\sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} v^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(u, v) - \sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} u^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(v, u) = 0$$

$$\sum_{\widehat{\mathcal{O}}} b^2_{\phi\widehat{\mathcal{O}}} f^{(d)}_{\widehat{\mathcal{O}}}(\xi, \eta) - \sum_{\mathcal{O}} \lambda_{\phi\phi\mathcal{O}} a_{\mathcal{O}} \xi^{-\Delta_\phi} f^{(b)}_{\mathcal{O}}(\xi, \eta) = 0$$

Want coefficients to be positive not sign indefinite.



[Liendo, Rastelli, van Rees; 1210.04258]
[Billo, Goncalves, Lauria, Meineri; 1601.02883]

$$\sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} v^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(u,v) - \sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} u^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(v,u) = 0$$

$$\sum_{\widehat{\mathcal{O}}} b^2_{\phi\widehat{\mathcal{O}}} f^{(d)}_{\widehat{\mathcal{O}}}(\xi,\eta) - \sum_{\mathcal{O}} \lambda_{\phi\phi\mathcal{O}} a_{\mathcal{O}} \xi^{-\Delta_\phi} f^{(b)}_{\mathcal{O}}(\xi,\eta) = 0$$

Want coefficients to be positive not sign indefinite.

1. Embed system into a matrix equation with positivity properties.

$$\sum_{\mathcal{O},\widehat{\mathcal{O}}} \begin{bmatrix} a_{\mathcal{O}} & b_{\phi\widehat{\mathcal{O}}} & \lambda_{\phi\phi\mathcal{O}} \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{bmatrix} \begin{bmatrix} a_{\mathcal{O}} \\ b_{\phi\widehat{\mathcal{O}}} \\ \lambda_{\phi\phi\mathcal{O}} \end{bmatrix} = C?$$

[Liendo, Rastelli, van Rees; 1210.04258]
[Billo, Goncalves, Lauria, Meineri; 1601.02883]

$$\sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} v^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(u,v) - \sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} u^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(v,u) = 0$$

$$\sum_{\widehat{\mathcal{O}}} b^2_{\phi\widehat{\mathcal{O}}} f^{(d)}_{\widehat{\mathcal{O}}}(\xi,\eta) - \sum_{\mathcal{O}} \lambda_{\phi\phi\mathcal{O}} a_{\mathcal{O}} \xi^{-\Delta_\phi} f^{(b)}_{\mathcal{O}}(\xi,\eta) = 0$$

Want coefficients to be positive not sign indefinite.

1. Embed system into a matrix equation with positivity properties.

$$\sum_{\widehat{\mathcal{O}}} \begin{bmatrix} b_{T\widehat{\mathcal{O}}} & \lambda_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix} \begin{bmatrix} b_{T\widehat{\mathcal{O}}} \\ \lambda_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} \end{bmatrix} = C$$

[Levine, Paulos; 2305.07078]
[Meineri, Penedones, Spirig; 2305.11209]

The defect bootstrap

[Liendo, Rastelli, van Rees; 1210.04258]
[Billo, Goncalves, Lauria, Meineri; 1601.02883]

$$\sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} v^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(u, v) - \sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} u^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(v, u) = 0$$

$$\sum_{\widehat{\mathcal{O}}} b^2_{\phi\widehat{\mathcal{O}}} f^{(d)}_{\widehat{\mathcal{O}}}(\xi, \eta) - \sum_{\mathcal{O}} \lambda_{\phi\phi\mathcal{O}} a_{\mathcal{O}} \xi^{-\Delta_\phi} f^{(b)}_{\mathcal{O}}(\xi, \eta) = 0$$

Want coefficients to be positive not sign indefinite.

1. Embed system into a matrix equation with positivity properties.

$$\sum_{\widehat{\mathcal{O}}} \begin{bmatrix} b_{T\widehat{\mathcal{O}}} & \lambda_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix} \begin{bmatrix} b_{T\widehat{\mathcal{O}}} \\ \lambda_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} \end{bmatrix} = C$$

[Levine, Paulos; 2305.07078]
[Meineri, Penedones, Spirig; 2305.11209]

$\langle T^{\mu\nu} T^{\rho\sigma} \rangle$ gives C_T in terms of $b^2_{T\widehat{\mathcal{O}}}$, $\langle T^{\mu\nu} \hat{\phi}\hat{\phi} \rangle$ gives Δ_ϕ in terms of $b_{T\widehat{\mathcal{O}}} \lambda_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}}$, $\langle \hat{\phi}\hat{\phi}\hat{\phi}\hat{\phi} \rangle$ completes system.

The defect bootstrap

[Liendo, Rastelli, van Rees; 1210.04258]
[Billo, Goncalves, Lauria, Meineri; 1601.02883]

$$\sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} v^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(u,v) - \sum_{\widehat{\mathcal{O}}} \lambda^2_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} u^{\widehat{\Delta}_\phi} g_{\widehat{\mathcal{O}}}(v,u) = 0$$

$$\sum_{\widehat{\mathcal{O}}} b^2_{\phi\widehat{\mathcal{O}}} f^{(d)}_{\widehat{\mathcal{O}}}(\xi,\eta) - \sum_{\mathcal{O}} \lambda_{\phi\phi\mathcal{O}} a_{\mathcal{O}} \xi^{-\Delta_\phi} f^{(b)}_{\mathcal{O}}(\xi,\eta) = 0$$

Want coefficients to be positive not sign indefinite.

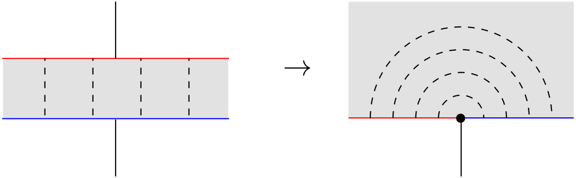
1. Embed system into a matrix equation with positivity properties.

$$\sum_{\widehat{\mathcal{O}}} \begin{bmatrix} b_{T\widehat{\mathcal{O}}} & \lambda_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix} \begin{bmatrix} b_{T\widehat{\mathcal{O}}} \\ \lambda_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}} \end{bmatrix} = C$$

[Levine, Paulos; 2305.07078]
[Meineri, Penedones, Spirig; 2305.11209]

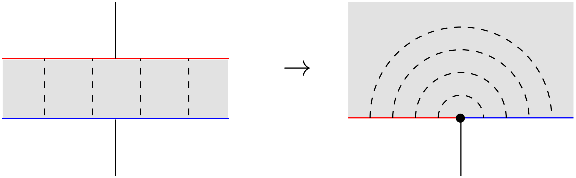
$\langle T^{\mu\nu} T^{\rho\sigma} \rangle$ gives C_T in terms of $b^2_{T\widehat{\mathcal{O}}}$, $\langle T^{\mu\nu} \hat{\phi}\hat{\phi} \rangle$ gives Δ_ϕ in terms of $b_{T\widehat{\mathcal{O}}} \lambda_{\hat{\phi}\hat{\phi}\widehat{\mathcal{O}}}$, $\langle \hat{\phi}\hat{\phi}\hat{\phi}\hat{\phi} \rangle$ completes system. For boundaries in 2d, see [Collier, Mazac, Wang; 2112.00750] [Meineri, Radhakrishnan; ?] .

2. Focus on the defect but use “defect changing operators”.



The defect bootstrap

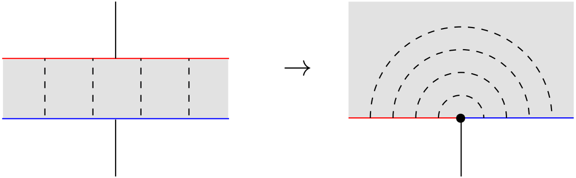
2. Focus on the defect but use "defect changing operators".



Let σ_{TN}, σ_{NT} switch between trivial and non-trivial line defects.

The defect bootstrap

2. Focus on the defect but use “defect changing operators”.

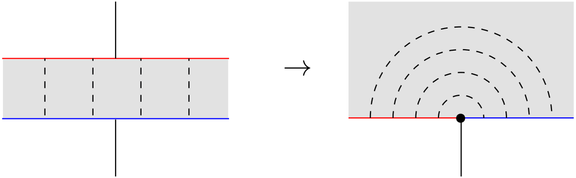


Let σ_{TN}, σ_{NT} switch between trivial and non-trivial line defects.

$$\left\langle \underbrace{\sigma_{TN}(\vec{x}_1)\sigma_{NT}(\vec{x}_2)} \widehat{\mathcal{O}}(\vec{x}_3)\widehat{\mathcal{O}}(\vec{x}_4) \right\rangle$$

Can study this OPE since defect operators $\widehat{\mathcal{O}}(\vec{x}_i)$ are really bulk operators with no defect! [Zhou, Gaiotto, He, Zou; 2401.00039] [Lanzetta, Liu, Metlitski; ?]

2. Focus on the defect but use “defect changing operators”.



Let σ_{TN}, σ_{NT} switch between trivial and non-trivial line defects.

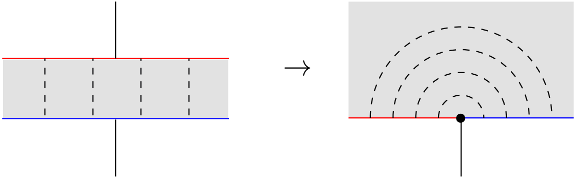
$$\left\langle \underbrace{\sigma_{TN}(\vec{x}_1)\sigma_{NT}(\vec{x}_2)} \widehat{\mathcal{O}}(\vec{x}_3)\widehat{\mathcal{O}}(\vec{x}_4) \right\rangle$$

Can study this OPE since defect operators $\widehat{\mathcal{O}}(\vec{x}_i)$ are really bulk operators with no defect! [Zhou, Gaiotto, He, Zou; 2401.00039] [Lanzetta, Liu, Metlitski; ?]

3. Focus on the defect but input bulk equations of motion.

$$\partial^x_\mu T^{\mu\nu}(\vec{x}, y) = -\partial^y_\mu T^{\mu\nu}(\vec{x}, y) \neq 0, \quad \widehat{\Delta}_T = d \neq p$$

2. Focus on the defect but use "defect changing operators".



Let σ_{TN}, σ_{NT} switch between trivial and non-trivial line defects.

$$\left\langle \underbrace{\sigma_{TN}(\vec{x}_1)\sigma_{NT}(\vec{x}_2)} \widehat{\mathcal{O}}(\vec{x}_3)\widehat{\mathcal{O}}(\vec{x}_4) \right\rangle$$

Can study this OPE since defect operators $\widehat{\mathcal{O}}(\vec{x}_i)$ are really bulk operators with no defect! [Zhou, Gaiotto, He, Zou; 2401.00039] [Lanzetta, Liu, Metlitski; ?]

3. Focus on the defect but input bulk equations of motion.

$$\partial^x_\mu T^{\mu\nu}(\vec{x}, y) = -\partial^y_\mu T^{\mu\nu}(\vec{x}, y) \neq 0, \quad \widehat{\Delta}_T = d \neq p$$

Has been explored in free scalar theory (this talk) and Maxwell theory: [Herzog, Shrestha; 2202.09180] [Bartlett-Tisdall, Herzog, Schaub; 2312.07692]

Free boson near a defect

$\Box\phi(\vec{x}, y) = 0$ has two solutions for each $SO(q)$ spin s.

$$\phi(\vec{x}, y) = \sum_{s=0}^{\infty} y_{i_1} \dots y_{i_s} \left[b_+^{(s)} |y|^s \hat{\psi}_+^{i_1 \dots i_s}(\vec{x}) + b_-^{(s)} |y|^{2-q-s} \hat{\psi}_-^{i_1 \dots i_s}(\vec{x}) \right] + \dots$$

Free boson near a defect

$\Box\phi(\vec{x}, y) = 0$ has two solutions for each $SO(q)$ spin s.

$$\phi(\vec{x}, y) = \sum_{s=0}^{\infty} y_{i_1} \dots y_{i_s} \left[b_+^{(s)} |y|^s \hat{\psi}_+^{i_1 \dots i_s}(\vec{x}) + b_-^{(s)} |y|^{2-q-s} \hat{\psi}_-^{i_1 \dots i_s}(\vec{x}) \right] + \dots$$

Special operators:

$$\widehat{\Delta}_+^{(s)} = \frac{d-2}{2} + s$$
$$\widehat{\Delta}_-^{(s)} = p - \frac{d-2}{2} - s$$

Free boson near a defect

$\Box\phi(\vec{x}, y) = 0$ has two solutions for each $SO(q)$ spin s.

$$\phi(\vec{x}, y) = \sum_{s=0}^{\infty} y_{i_1} \dots y_{i_s} \left[b_+^{(s)} |y|^s \hat{\psi}_+^{i_1 \dots i_s}(\vec{x}) + b_-^{(s)} |y|^{2-q-s} \hat{\psi}_-^{i_1 \dots i_s}(\vec{x}) \right] + \dots$$

Special operators:

$$\widehat{\Delta}_+^{(s)} = \tfrac{d-2}{2} + s$$
$$\widehat{\Delta}_-^{(s)} = p - \tfrac{d-2}{2} - s$$

Boundary ($q = 1, s = 0$) example:

$$\hat{\psi}_+^{(0)} = \hat{\phi}, \qquad D: \; b_+^{(0)} = 0$$
$$\hat{\psi}_-^{(0)} = \partial_y \hat{\phi}, \qquad N: \; b_-^{(0)} = 0$$

Free boson near a defect

$\Box\phi(\vec{x}, y) = 0$ has two solutions for each $SO(q)$ spin s.

$$\phi(\vec{x}, y) = \sum_{s=0}^{\infty} y_{i_1} \dots y_{i_s} \left[b_+^{(s)} |y|^s \hat{\psi}_+^{i_1 \dots i_s}(\vec{x}) + b_-^{(s)} |y|^{2-q-s} \hat{\psi}_-^{i_1 \dots i_s}(\vec{x}) \right] + \dots$$

Special operators:

$$\widehat{\Delta}_+^{(s)} = \tfrac{d-2}{2} + s$$
$$\widehat{\Delta}_-^{(s)} = p - \tfrac{d-2}{2} - s$$

Boundary ($q = 1, s = 0$) example:

$$\hat{\psi}_+^{(0)} = \hat{\phi}, \qquad D: \; b_+^{(0)} = 0$$
$$\hat{\psi}_-^{(0)} = \partial_y \hat{\phi}, \qquad N: \; b_-^{(0)} = 0$$

Starting with D or N, we can couple to any CFT_p with relevant $\hat{\Phi}$.

$$S_{int} = g \int_{\mathbb{R}^p} \hat{\psi}_\pm^{(0)} \hat{\Phi} \, d^p\vec{x}, \quad \partial_\mu T^{\mu\nu} \propto g \psi_+^{(0)} \partial^\nu \psi_-^{(0)}$$

Free boson near a defect

$\Box\phi(\vec{x}, y) = 0$ has two solutions for each $SO(q)$ spin s.

$$\phi(\vec{x}, y) = \sum_{s=0}^{\infty} y_{i_1} \dots y_{i_s} \left[b_+^{(s)} |y|^s \hat{\psi}_+^{i_1 \dots i_s}(\vec{x}) + b_-^{(s)} |y|^{2-q-s} \hat{\psi}_-^{i_1 \dots i_s}(\vec{x}) \right] + \dots$$

Special operators:

$$\widehat{\Delta}_+^{(s)} = \tfrac{d-2}{2} + s$$
$$\widehat{\Delta}_-^{(s)} = p - \tfrac{d-2}{2} - s$$

Boundary $(q = 1, s = 0)$ example:

$$\hat{\psi}_+^{(0)} = \hat{\phi}, \qquad D: \; b_+^{(0)} = 0$$
$$\hat{\psi}_-^{(0)} = \partial_y \hat{\phi}, \qquad N: \; b_-^{(0)} = 0$$

Starting with D or N, we can couple to any CFT_p with relevant $\hat{\Phi}$.

$$S_{int} = g \int_{\mathbb{R}^p} \hat{\psi}_\pm^{(0)} \hat{\Phi} \, d^p \vec{x}, \quad \partial_\mu T^{\mu\nu} \propto g \psi_+^{(0)} \partial^\nu \psi_-^{(0)}$$

Key relations for bootstrapping $b_\pm^{(s)} \neq 0$ defects:

$\lambda_{++\widehat{\mathcal{O}}} = \kappa_+(\widehat{\Delta}, \ell) \lambda_{+-\widehat{\mathcal{O}}}$ and $\lambda_{--\widehat{\mathcal{O}}} = \kappa_-(\widehat{\Delta}, \ell) \lambda_{+-\widehat{\mathcal{O}}}$

OPE relations in a different context

Consider long range Ising model $H = -J \sum_{i,j} \sigma_i \sigma_j / |i-j|^{d+\mathfrak{s}}$.

Action	Nonlocal EOM
$S = \int \frac{\phi(x)\phi(y)}{\lvert x-y\rvert^{d+\mathfrak{s}}} d^d x d^d y + \lambda \int \phi^4 d^d x$	$\phi^3(x) = \int \frac{\phi(y) d^d y}{\lvert x-y\rvert^{d+\mathfrak{s}}}$
[Fisher, Ma, Nickel; 72]	$\Delta_\phi + \Delta_{\phi^3} = d$

OPE relations in a different context

Consider long range Ising model $H = -J \sum_{i,j} \sigma_i \sigma_j / |i-j|^{d+\mathfrak{s}}$.

Action	Nonlocal EOM
$S = \int \frac{\phi(x)\phi(y)}{\|x-y\|^{d+\mathfrak{s}}} d^d x d^d y + \lambda \int \phi^4 d^d x$	$\phi^3(x) = \int \frac{\phi(y) d^d y}{\|x-y\|^{d+\mathfrak{s}}}$
[Fisher, Ma, Nickel; 72]	$\Delta_\phi + \Delta_{\phi^3} = d$
$S = S_{SRI} + \int \frac{\chi(x)\chi(y)}{\|x-y\|^{d-\mathfrak{s}}} d^d x d^d y + g \int \sigma \chi d^d x$	$\sigma(x) = \int \frac{\chi(y) d^d y}{\|x-y\|^{d-\mathfrak{s}}}$
[CB, Rastelli, Rychkov, Zan; 1703.05325]	$\Delta_\sigma + \Delta_\chi = d$

OPE relations in a different context

Consider long range Ising model $H = -J \sum_{i,j} \sigma_i \sigma_j / |i-j|^{d+\mathfrak{s}}$.

Action (same fixed point for both!)	Nonlocal EOM
$S = \int \frac{\phi(x)\phi(y)}{\|x-y\|^{d+\mathfrak{s}}} d^dx d^dy + \lambda \int \phi^4 d^dx$	$\phi^3(x) = \int \frac{\phi(y) d^dy}{\|x-y\|^{d+\mathfrak{s}}}$
[Fisher, Ma, Nickel; 72]	$\Delta_\phi + \Delta_{\phi^3} = d$
$S = S_{SRI} + \int \frac{\chi(x)\chi(y)}{\|x-y\|^{d-\mathfrak{s}}} d^dx d^dy + g \int \sigma\chi d^dx$	$\sigma(x) = \int \frac{\chi(y) d^dy}{\|x-y\|^{d-\mathfrak{s}}}$
[CB, Rastelli, Rychkov, Zan; 1703.05325]	$\Delta_\sigma + \Delta_\chi = d$

OPE relations in a different context

Consider long range Ising model $H = -J \sum_{i,j} \sigma_i \sigma_j / |i-j|^{d+\mathfrak{s}}$.

Action (same fixed point for both!)	Nonlocal EOM
$S = \int \frac{\phi(x)\phi(y)}{\lvert x-y\rvert^{d+\mathfrak{s}}} d^d x d^d y + \lambda \int \phi^4 d^d x$	$\phi^3(x) = \int \frac{\phi(y) d^d y}{\lvert x-y\rvert^{d+\mathfrak{s}}}$
[Fisher, Ma, Nickel; 72]	$\Delta_\phi + \Delta_{\phi^3} = d$
$S = S_{SRI} + \int \frac{\chi(x)\chi(y)}{\lvert x-y\rvert^{d-\mathfrak{s}}} d^d x d^d y + g \int \sigma\chi d^d x$	$\sigma(x) = \int \frac{\chi(y) d^d y}{\lvert x-y\rvert^{d-\mathfrak{s}}}$
[CB, Rastelli, Rychkov, Zan; 1703.05325]	$\Delta_\sigma + \Delta_\chi = d$

Can derive $\lambda_{\sigma\sigma\mathcal{O}} = \kappa_\sigma(\Delta, \ell)\lambda_{\sigma\chi\mathcal{O}}$ and $\lambda_{\chi\chi\mathcal{O}} = \kappa_\chi(\Delta, \ell)\lambda_{\sigma\chi\mathcal{O}}$ since $\langle\sigma\chi\mathcal{O}\rangle$ and $\langle\chi\chi\mathcal{O}\rangle$ are shadow integral transforms of each other [Paulos, Rychkov, van Rees, Zan; 1509.00008] [CB; 1810.07199] .

OPE relations in a different context

Consider long range Ising model $H = -J\sum_{i,j}\sigma_i\sigma_j/|i-j|^{p+\mathfrak{s}}$.

Action (same fixed point for both!)	Nonlocal EOM
$S = \int \frac{\hat{\phi}(\vec{x})\hat{\phi}(\vec{y})}{\|\vec{x}-\vec{y}\|^{p+\mathfrak{s}}} d^p\vec{x}d^p\vec{y} + \lambda\int\hat{\phi}^4 d^p\vec{x}$	$\hat{\phi}^3(\vec{x}) = \int \frac{\hat{\phi}(\vec{y})d^p\vec{y}}{\|\vec{x}-\vec{y}\|^{p+\mathfrak{s}}}$
[Fisher, Ma, Nickel; 72]	$\widehat{\Delta}_\phi + \widehat{\Delta}_{\phi^3} = p$
$S = S_{SRI} + \int \frac{\hat{\chi}(\vec{x})\hat{\chi}(\vec{y})}{\|\vec{x}-\vec{y}\|^{p-\mathfrak{s}}} d^p\vec{x}d^p\vec{y} + g\int\hat{\sigma}\hat{\chi}d^p\vec{x}$	$\hat{\sigma}(\vec{x}) = \int \frac{\hat{\chi}(\vec{y})d^p\vec{y}}{\|\vec{x}-\vec{y}\|^{p-\mathfrak{s}}}$
[CB, Rastelli, Rychkov, Zan; 1703.05325]	$\widehat{\Delta}_\sigma + \widehat{\Delta}_\chi = p$

Can derive $\lambda_{\hat{\sigma}\hat{\sigma}\widehat{\mathcal{O}}} = \kappa_\sigma(\widehat{\Delta},\ell)\lambda_{\hat{\sigma}\hat{\chi}\widehat{\mathcal{O}}}$ and $\lambda_{\hat{\chi}\hat{\chi}\widehat{\mathcal{O}}} = \kappa_\chi(\widehat{\Delta},\ell)\lambda_{\hat{\sigma}\hat{\chi}\widehat{\mathcal{O}}}$ since $\langle\hat{\sigma}\hat{\chi}\widehat{\mathcal{O}}\rangle$ and $\langle\hat{\chi}\hat{\chi}\widehat{\mathcal{O}}\rangle$ are shadow integral transforms of each other [Paulos, Rychkov, van Rees, Zan; 1509.00008] [CB; 1810.07199] .

$$\hat{\phi} = \hat{\psi}_+^{(0)}, \qquad \hat{\phi}^3 = \hat{\psi}_-^{(0)}, \qquad q = 2-\mathfrak{s}$$

$$\hat{\chi} = \hat{\psi}_+^{(0)}, \qquad \hat{\sigma} = \hat{\psi}_-^{(0)}, \qquad q = 2+\mathfrak{s}$$

General OPE relations

$$\frac{\lambda_{++\widehat{\mathcal{O}}}}{\lambda_{-+\widehat{\mathcal{O}}}} = R\frac{\Gamma[\frac{1}{2}(\ell + \widehat{\Delta})]\Gamma[\frac{1}{2}(\ell + p + q - 2 - \widehat{\Delta})]}{\Gamma[\frac{1}{2}(\ell + 2 - q + \widehat{\Delta})]\Gamma[\frac{1}{2}(\ell + p - \widehat{\Delta})]}$$

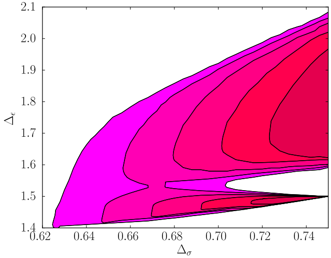
$$\frac{\lambda_{--\widehat{\mathcal{O}}}}{\lambda_{+-\widehat{\mathcal{O}}}} = R^{-1}\frac{\Gamma[\frac{1}{2}(\ell + \widehat{\Delta})]\Gamma[\frac{1}{2}(\ell + p - q + 2 - \widehat{\Delta})]}{\Gamma[\frac{1}{2}(\ell - 2 + q + \widehat{\Delta})]\Gamma[\frac{1}{2}(\ell + p - \widehat{\Delta})]}$$

General OPE relations

$$\frac{\lambda_{++\widehat{\mathcal{O}}}}{\lambda_{-+\widehat{\mathcal{O}}}} = R\frac{\Gamma[\frac{1}{2}(\ell + \widehat{\Delta})]\Gamma[\frac{1}{2}(\ell + p + q - 2 - \widehat{\Delta})]}{\Gamma[\frac{1}{2}(\ell + 2 - q + \widehat{\Delta})]\Gamma[\frac{1}{2}(\ell + p - \widehat{\Delta})]}$$

$$\frac{\lambda_{--\widehat{\mathcal{O}}}}{\lambda_{+-\widehat{\mathcal{O}}}} = R^{-1}\frac{\Gamma[\frac{1}{2}(\ell + \widehat{\Delta})]\Gamma[\frac{1}{2}(\ell + p - q + 2 - \widehat{\Delta})]}{\Gamma[\frac{1}{2}(\ell - 2 + q + \widehat{\Delta})]\Gamma[\frac{1}{2}(\ell + p - \widehat{\Delta})]}$$

[CB; 1810.07199]



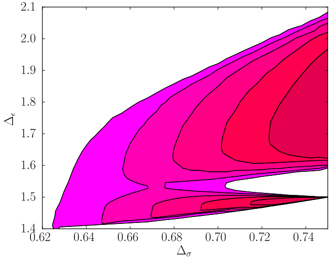
General OPE relations

$$\frac{\lambda_{++\widehat{\mathcal{O}}}}{\lambda_{-+\widehat{\mathcal{O}}}} = R\frac{\Gamma[\frac{1}{2}(\ell+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell+p+q-2-\widehat{\Delta})]}{\Gamma[\frac{1}{2}(\ell+2-q+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell+p-\widehat{\Delta})]}$$

$$\frac{\lambda_{--\widehat{\mathcal{O}}}}{\lambda_{+-\widehat{\mathcal{O}}}} = R^{-1}\frac{\Gamma[\frac{1}{2}(\ell+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell+p-q+2-\widehat{\Delta})]}{\Gamma[\frac{1}{2}(\ell-2+q+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell+p-\widehat{\Delta})]}$$

$$R = -\frac{b_-^{(0)}\Gamma[(4-q)/2]}{b_+^{(0)}\Gamma[q/2]}$$

[CB; 1810.07199]



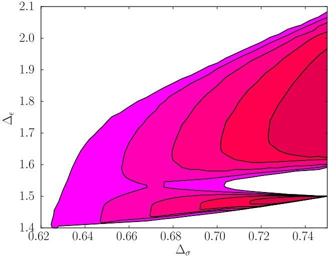
General OPE relations

$$\frac{\lambda^{s_1,s_2,-s_1-s_2}_{++\widehat{\mathcal{O}}}}{\lambda^{s_1,s_2,-s_1-s_2}_{-+\widehat{\mathcal{O}}}} = R\frac{\Gamma[\frac{1}{2}(\ell+s_{12}+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell+s_1+s_2+p+q-2-\widehat{\Delta})]}{\Gamma[\frac{1}{2}(\ell-s_1-s_2+2-q+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell-s_{12}+p-\widehat{\Delta})]}$$

$$\frac{\lambda^{s_1,s_2,-s_1-s_2}_{--\widehat{\mathcal{O}}}}{\lambda^{s_1,s_2,-s_1-s_2}_{+-\widehat{\mathcal{O}}}} = R^{-1}\frac{\Gamma[\frac{1}{2}(\ell-s_{12}+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell-s_1-s_2+p-q+2-\widehat{\Delta})]}{\Gamma[\frac{1}{2}(\ell+s_1+s_2-2+q+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell+s_{12}+p-\widehat{\Delta})]}$$

$$R = -\frac{b_-^{(s_1)}\Gamma[(4-q)/2-s_1]}{b_+^{(s_1)}\Gamma[q/2+s_1]}$$

[CB; 1810.07199]



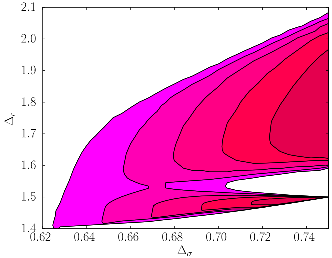
General OPE relations

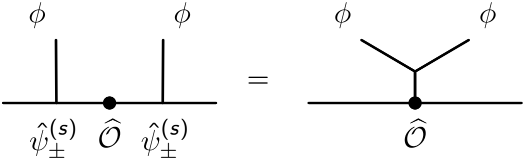
$$\frac{\lambda^{s_1,s_2,-s_1-s_2}_{++\widehat{\mathcal{O}}}}{\lambda^{s_1,s_2,-s_1-s_2}_{-+\widehat{\mathcal{O}}}} = R\frac{\Gamma[\frac{1}{2}(\ell+s_{12}+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell+s_1+s_2+p+q-2-\widehat{\Delta})]}{\Gamma[\frac{1}{2}(\ell-s_1-s_2+2-q+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell-s_{12}+p-\widehat{\Delta})]}$$

$$\frac{\lambda^{s_1,s_2,-s_1-s_2}_{--\widehat{\mathcal{O}}}}{\lambda^{s_1,s_2,-s_1-s_2}_{+-\widehat{\mathcal{O}}}} = R^{-1}\frac{\Gamma[\frac{1}{2}(\ell-s_{12}+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell-s_1-s_2+p-q+2-\widehat{\Delta})]}{\Gamma[\frac{1}{2}(\ell+s_1+s_2-2+q+\widehat{\Delta})]\Gamma[\frac{1}{2}(\ell+s_{12}+p-\widehat{\Delta})]}$$

$$R = -\frac{b_-^{(s_1)}\Gamma[(4-q)/2-s_1]}{b_+^{(s_1)}\Gamma[q/2+s_1]}$$

[CB; 1810.07199]





Right side is regular so we get constraints on the left side.

Targets

Most $\hat{\psi}^{(s)}_{-}(\vec{x})$ are non-unitary ($s \geq \frac{4-q}{2}$).

Only $\hat{\psi}^{(s)}_{+}(\vec{x})$ makes the defect trivial [Lauria, Liendo, van Rees, Zhao; 2005.02413] .

Targets

Most $\hat{\psi}^{(s)}_-(\vec{x})$ are non-unitary ($s \geq \frac{4-q}{2}$).
Only $\hat{\psi}^{(s)}_+(\vec{x})$ makes the defect trivial [Lauria, Liendo, van Rees, Zhao; 2005.02413] .

Allowed cases (besides fractional):

$$q = 1, s = 0$$
$$q = 2, s = \frac{1}{2} \ (\textit{monodromy})$$
$$q = 3, s = 0$$

Targets

Most $\hat{\psi}^{(s)}_-(\vec{x})$ are non-unitary ($s \geq \frac{4-q}{2}$).

Only $\hat{\psi}^{(s)}_+(\vec{x})$ makes the defect trivial [Lauria, Liendo, van Rees, Zhao; 2005.02413] .

Allowed cases (besides fractional):

$$q = 1, s = 0$$
$$q = 2, s = \frac{1}{2} \; (\textit{monodromy})$$
$$q = 3, s = 0$$

Start with $q = 1$:

$$b_+^2 = 1 + 2^{d-2} a_{\phi^2}$$
$$b_-^2 = (d-2)\left(1 - 2^{d-2} a_{\phi^2}\right)$$
$$-2^{2-d} \leq a_{\phi^2} \leq 2^{2-d}$$

Targets

Most $\hat{\psi}_-^{(s)}(\vec{x})$ are non-unitary ($s \geq \frac{4-q}{2}$).

Only $\hat{\psi}_+^{(s)}(\vec{x})$ makes the defect trivial [Lauria, Liendo, van Rees, Zhao; 2005.02413] .

Allowed cases (besides fractional):

$$q = 1, s = 0$$
$$q = 2, s = \frac{1}{2} \ (\textit{monodromy})$$
$$q = 3, s = 0$$

Start with $q = 1$:

$$b_+^2 = 1 + 2^{d-2} a_{\phi^2}$$
$$b_-^2 = (d-2)\left(1 - 2^{d-2} a_{\phi^2}\right)$$
$$-2^{2-d} \leq a_{\phi^2} \leq 2^{2-d}$$

Bootstrap $\left\langle \hat{\psi}_+\hat{\psi}_+\hat{\psi}_+\hat{\psi}_+ \right\rangle$, $\left\langle \hat{\psi}_-\hat{\psi}_-\hat{\psi}_-\hat{\psi}_- \right\rangle$, $\left\langle \hat{\psi}_+\hat{\psi}_+\hat{\psi}_-\hat{\psi}_- \right\rangle$ where OPE relations reduce $\left\{\lambda_{++\widehat{\mathcal{O}}}, \lambda_{+-\widehat{\mathcal{O}}}, \lambda_{--\widehat{\mathcal{O}}}\right\} \to \left\{\lambda_{+-\widehat{\mathcal{O}}}\right\}$.

Targets

Most $\hat{\psi}_-^{(s)}(\vec{x})$ are non-unitary ($s \geq \frac{4-q}{2}$).
Only $\hat{\psi}_+^{(s)}(\vec{x})$ makes the defect trivial [Lauria, Liendo, van Rees, Zhao; 2005.02413] .

Allowed cases (besides fractional):

$$q = 1, s = 0$$
$$q = 2, s = \frac{1}{2} \ \textcolor{red}{(monodromy)}$$
$$q = 3, s = 0$$

Start with $q = 1$:

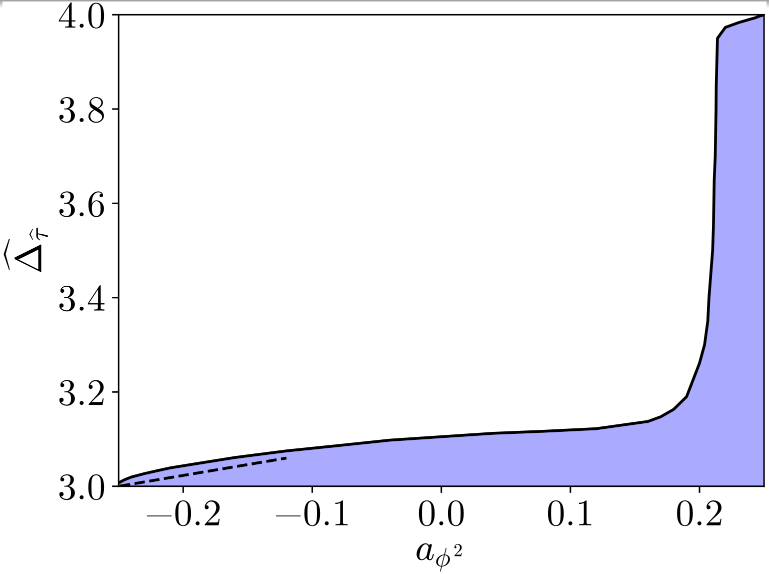
$$b_+^2 = 1 + 2^{d-2} a_{\phi^2}$$
$$b_-^2 = (d-2)\left(1 - 2^{d-2} a_{\phi^2}\right)$$
$$-2^{2-d} \leq a_{\phi^2} \leq 2^{2-d}$$

Bootstrap $\left\langle \hat{\psi}_+\hat{\psi}_+\hat{\psi}_+\hat{\psi}_+ \right\rangle$, $\left\langle \hat{\psi}_-\hat{\psi}_-\hat{\psi}_-\hat{\psi}_- \right\rangle$, $\left\langle \hat{\psi}_+\hat{\psi}_+\hat{\psi}_-\hat{\psi}_- \right\rangle$ where OPE relations reduce $\left\{\lambda_{++\hat{\mathcal{O}}}, \lambda_{+-\hat{\mathcal{O}}}, \lambda_{--\hat{\mathcal{O}}}\right\} \to \left\{\lambda_{+-\hat{\mathcal{O}}}\right\}$.

Odd spin: If $\hat{\Delta} \neq d - 1 + 2n + \ell$ ($\hat{\mathcal{O}} \neq [\psi_+\psi_-]_{n,\ell}$) then $\lambda_{**\hat{\mathcal{O}}} = 0$
Even spin: If $\hat{\Delta} = d - 2 + 2n + \ell$ ($\hat{\mathcal{O}} = [\psi_+\psi_+]_{n,\ell}, [\psi_-\psi_-]_{n,\ell}$) then $\lambda_{+-\hat{\mathcal{O}}} = 0$ while $\lambda_{++\hat{\mathcal{O}}}$ and $\lambda_{--\hat{\mathcal{O}}}$ are unconstrained

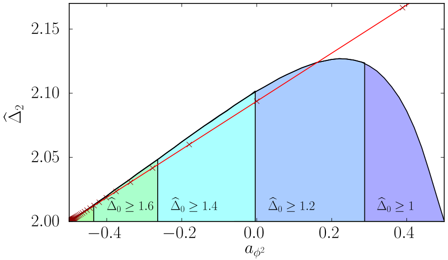
[CB, Di Pietro, Lauria, van Rees; 2009.03336]

Results, 4d



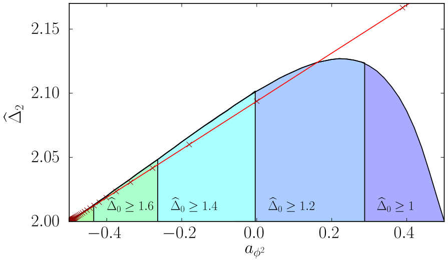
Maximizing the gap for spin-2 operators from left (Dirichlet) to right (Neumann) [CB, Di Pietro, Lauria, van Rees; 2009.03336] .

Results, 3d



[CB, Di Pietro, Lauria, van Rees; 2111.04747]

Results, 3d



[CB, Di Pietro, Lauria, van Rees; 2111.04747]

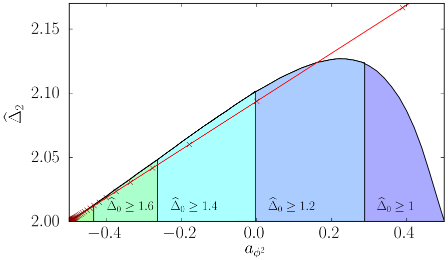
Use large m minimal model w/ $\widehat{\Delta}_{(1,2)} \sim \frac{1}{2} - \frac{3}{2m}$, $\widehat{\Delta}_{(1,3)} \sim 2 - \frac{4}{m}$ in

$$S_{int} = g \int_{\mathbb{R}^2} \hat{\psi}_- \hat{\Phi}_{(1,2)} d^2\vec{x} + h \int_{\mathbb{R}^2} \hat{\Phi}_{(1,3)} d^2\vec{x}.$$

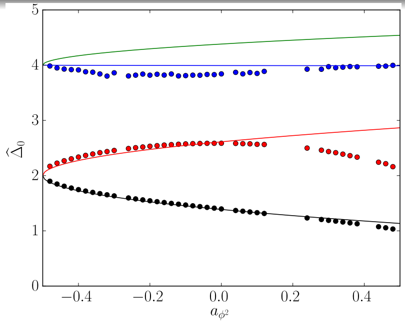
Plug fixed point into

$$\delta a_{\phi^2} = g^2 \pi^{d/2} \frac{2^{3-d}}{\Gamma[\frac{d}{2}]}, \quad \gamma_\tau = g^2 \pi^{d/2-1} \frac{d-1}{d+1} \frac{\Gamma[\frac{d}{2}+1]}{\Gamma[\frac{d+1}{2}]^2}.$$

Results, 3d



[CB, Di Pietro, Lauria, van Rees; 2111.04747]



Use large m minimal model w/ $\widehat{\Delta}_{(1,2)} \sim \frac{1}{2} - \frac{3}{2m}$, $\widehat{\Delta}_{(1,3)} \sim 2 - \frac{4}{m}$ in

$$S_{int} = g \int_{\mathbb{R}^2} \hat{\psi}_- \hat{\Phi}_{(1,2)} d^2\vec{x} + h \int_{\mathbb{R}^2} \hat{\Phi}_{(1,3)} d^2\vec{x}.$$

Plug fixed point into

$$\delta a_{\phi^2} = g^2 \pi^{d/2} \frac{2^{3-d}}{\Gamma[\frac{d}{2}]}, \quad \gamma_\tau = g^2 \pi^{d/2-1} \frac{d-1}{d+1} \frac{\Gamma[\frac{d}{2}+1]}{\Gamma[\frac{d+1}{2}]^2}.$$

The displacement operator

Displacement $\hat{D}(\vec{x}) \equiv T_{\perp\perp}(\vec{x}, 0)$ is a protected $\widehat{\Delta} = d$ scalar.
Normalization $\left\langle \hat{D}(\vec{x})\hat{D}(0) \right\rangle = C_D/|\vec{x}|^{2d}$ guarantees

$$\int_{\mathbb{R}^{d-1}} \left\langle \phi(x_1)\phi(x_2)\hat{D}(\vec{x}) \right\rangle d^{d-1}\vec{x} = (\partial_{y_1} + \partial_{y_2}) \left\langle \phi(x_1)\phi(x_2) \right\rangle .$$

The displacement operator

Displacement $\hat{D}(\vec{x}) \equiv T_{\perp\perp}(\vec{x}, 0)$ is a protected $\widehat{\Delta} = d$ scalar.

Normalization $\left\langle \hat{D}(\vec{x}) \hat{D}(0) \right\rangle = C_D / |\vec{x}|^{2d}$ guarantees

$$\int_{\mathbb{R}^{d-1}} \left\langle \phi(x_1)\phi(x_2)\hat{D}(\vec{x}) \right\rangle d^{d-1}\vec{x} = (\partial_{y_1} + \partial_{y_2}) \left\langle \phi(x_1)\phi(x_2) \right\rangle .$$

Use bulk-defect crossing for this correlator to derive

$$\frac{\lambda_{++D}}{d-2} = \frac{2C_D S_d^2 + 2^d a_{\phi^2}}{4(d-1)S_d b_+^2}, \quad \frac{\lambda_{--D}}{d-2} = \frac{2C_D S_d^2 - 2^d a_{\phi^2}}{2S_d b_-^2}.$$

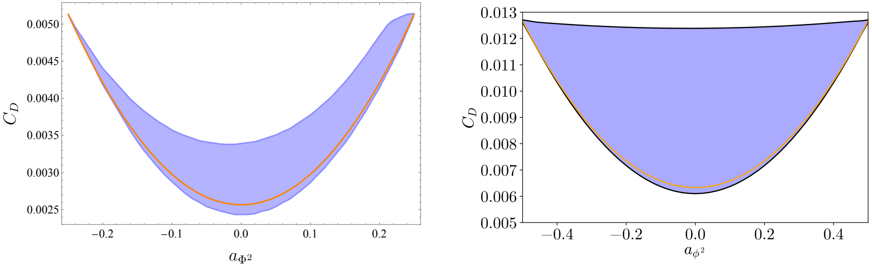
The displacement operator

Displacement $\hat{D}(\vec{x}) \equiv T_{\perp\perp}(\vec{x}, 0)$ is a protected $\widehat{\Delta} = d$ scalar.
Normalization $\left\langle \hat{D}(\vec{x})\hat{D}(0) \right\rangle = C_D/|\vec{x}|^{2d}$ guarantees

$$\int_{\mathbb{R}^{d-1}} \left\langle \phi(x_1)\phi(x_2)\hat{D}(\vec{x}) \right\rangle d^{d-1}\vec{x} = (\partial_{y_1} + \partial_{y_2}) \left\langle \phi(x_1)\phi(x_2) \right\rangle .$$

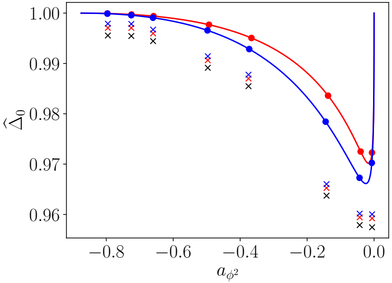
Use bulk-defect crossing for this correlator to derive

$$\frac{\lambda_{++D}}{d-2} = \frac{2C_D S_d^2 + 2^d a_{\phi^2}}{4(d-1)S_d b_+^2}, \qquad \frac{\lambda_{--D}}{d-2} = \frac{2C_D S_d^2 - 2^d a_{\phi^2}}{2S_d b_-^2}.$$



Lower edge is the solvable deformation in [Witten; hep-th/0112258] .

Back to long-range Ising



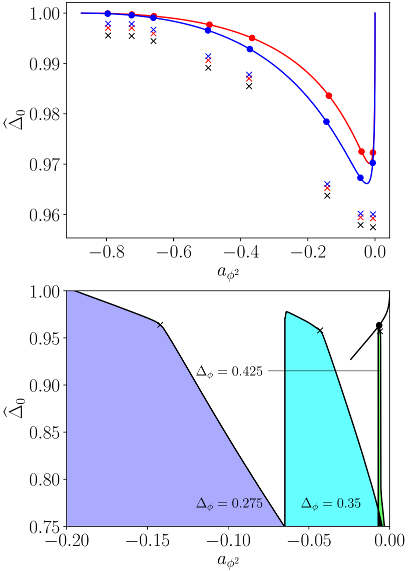
3-loop results from

[Benedetti, Gurau, Harribey, Suzuki; 2007.04603]

are close to numerical kinks from

[CB, Lauria, Nocchi, van Vliet; 2311.02742] .

Back to long-range Ising



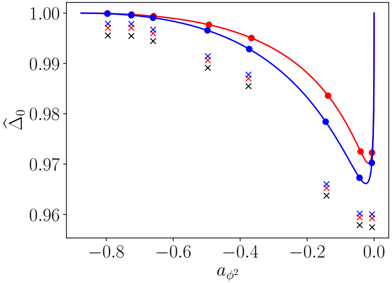
3-loop results from

[Benedetti, Gurau, Harribey, Suzuki; 2007.04603]

are close to numerical kinks from

[CB, Lauria, Nocchi, van Vliet; 2311.02742] .

Back to long-range Ising

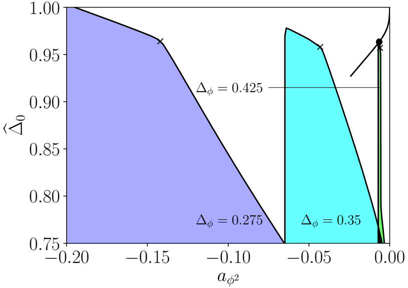


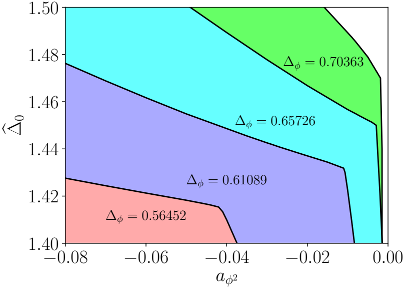
3-loop results from

[Benedetti, Gurau, Harribey, Suzuki; 2007.04603]

are close to numerical kinks from

[CB, Lauria, Nocchi, van Vliet; 2311.02742] .





More structure

LRI admits OPE relations and comes from $\hat{\phi}^4$ flow in $q = 2 - \mathfrak{s}$ and $\hat{\sigma}\hat{\chi}$ flow in $q = 2 + \mathfrak{s}$.

More structure

LRI admits OPE relations and comes from $\hat{\phi}^4$ flow in $q = 2 - \mathfrak{s}$ and $\hat{\sigma}\hat{\chi}$ flow in $q = 2 + \mathfrak{s}$.

If this duality holds generally, $q = 3$ will immediately follow from $q = 1$.

More structure

LRI admits OPE relations and comes from $\hat{\phi}^4$ flow in $q = 2 - \mathfrak{s}$ and $\hat{\sigma}\hat{\chi}$ flow in $q = 2 + \mathfrak{s}$.

If this duality holds generally, $q = 3$ will immediately follow from $q = 1$.

$$\widehat{\Delta}_{+}^{(0)} = \frac{p + q - 2}{2}, \quad \widehat{\Delta}_{-}^{(0)} = \frac{p + (4 - q) - 2}{2}$$

Switching $q \leftrightarrow 4 - q$, $\hat{\psi}_{+}^{(0)} \leftrightarrow \hat{\psi}_{-}^{(0)}$ leaves OPE relations invariant!

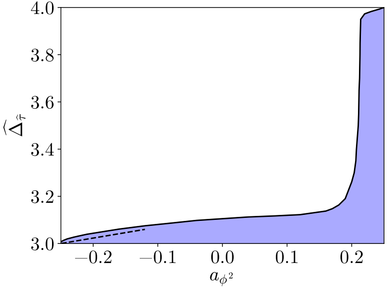
More structure

LRI admits OPE relations and comes from $\hat{\phi}^4$ flow in $q = 2 - \mathfrak{s}$ and $\hat{\sigma}\hat{\chi}$ flow in $q = 2 + \mathfrak{s}$.

If this duality holds generally, $q = 3$ will immediately follow from $q = 1$.

$$\widehat{\Delta}_+^{(0)} = \frac{p + q - 2}{2}, \quad \widehat{\Delta}_-^{(0)} = \frac{p + (4 - q) - 2}{2}$$

Switching $q \leftrightarrow 4 - q$, $\hat{\psi}_+^{(0)} \leftrightarrow \hat{\psi}_-^{(0)}$ leaves OPE relations invariant!

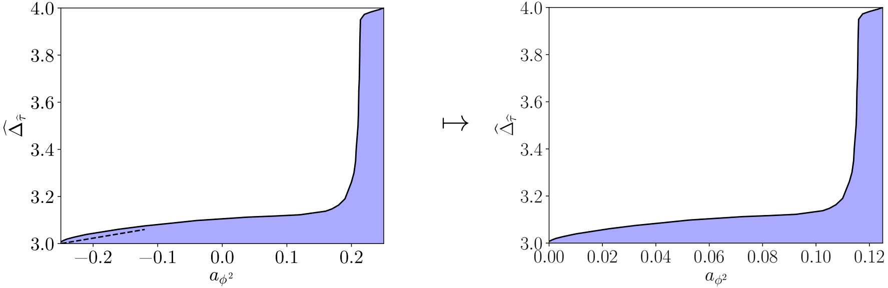


LRI admits OPE relations and comes from $\hat{\phi}^4$ flow in $q = 2 - \mathfrak{s}$ and $\hat{\sigma}\hat{\chi}$ flow in $q = 2 + \mathfrak{s}$.

If this duality holds generally, $q = 3$ will immediately follow from $q = 1$.

$$\hat{\Delta}_+^{(0)} = \frac{p+q-2}{2}, \quad \hat{\Delta}_-^{(0)} = \frac{p+(4-q)-2}{2}$$

Switching $q \leftrightarrow 4 - q$, $\hat{\psi}_+^{(0)} \leftrightarrow \hat{\psi}_-^{(0)}$ leaves OPE relations invariant!



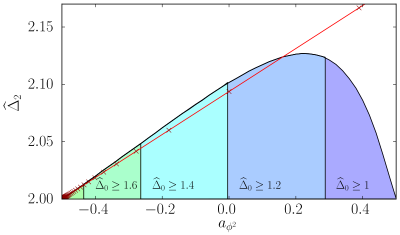
More structure

LRI admits OPE relations and comes from $\hat{\phi}^4$ flow in $q = 2 - \mathfrak{s}$ and $\hat{\sigma}\hat{\chi}$ flow in $q = 2 + \mathfrak{s}$.

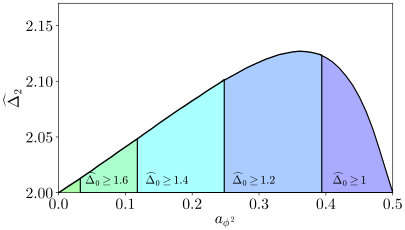
If this duality holds generally, $q = 3$ will immediately follow from $q = 1$.

$$\widehat{\Delta}^{(0)}_+ = \frac{p + q - 2}{2}, \quad \widehat{\Delta}^{(0)}_- = \frac{p + (4 - q) - 2}{2}$$

Switching $q \leftrightarrow 4 - q$, $\hat{\psi}^{(0)}_+ \leftrightarrow \hat{\psi}^{(0)}_-$ leaves OPE relations invariant!



$\mapsto$



Next steps

- Evidence of $q \leftrightarrow 4-q$ duality should also be visible from an analytic bootstrap point of view [Lemos, Liendo, Meineri, Sarkar; 1712.08185] [Liendo, Linke, Schomerus; 1903.05222] .
- Self-dual case $q = 2$ allows monodromy and should be bootstrapped [CB, Lauria, van Vliet; WIP] .
- OPE relations should exist for free fermion theory and analogous defect CFTs with SUSY [Bason, Di Pietro, Valandro, van Muiden; 2311.17888] .
- Unique numerical needs could benefit from more specialized tools [Ghosh, Zhang; 2307.11144] .

Next steps

- Evidence of $q \leftrightarrow 4 - q$ duality should also be visible from an analytic bootstrap point of view [Lemos, Liendo, Meineri, Sarkar; 1712.08185] [Liendo, Linke, Schomerus; 1903.05222] .
- Self-dual case $q = 2$ allows monodromy and should be bootstrapped [CB, Lauria, van Vliet; WIP] .
- OPE relations should exist for free fermion theory and analogous defect CFTs with SUSY [Bason, Di Pietro, Valandro, van Muiden; 2311.17888] .
- Unique numerical needs could benefit from more specialized tools [Ghosh, Zhang; 2307.11144] .

Thanks for your attention!