Protected operator algebras and holography

Connor Behan

Oxford Mathematical Institute

2021-03-24

Based on [2101.04114] with
Pietro Ferrero, Xinan Zhou

Basic playground

Superconformal field theories

↙ ↘

Supergravity

Chiral algebra or TQFT

[Maldacena; hep-th/9711200]

[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees; 1312.5344]

[Chester, Lee, Pufu, Yacoby; 1412.0334]

Basic playground

Superconformal field theories

↙ ↘

Supergravity

Chiral algebra or TQFT

[Maldacena; hep-th/9711200]

[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees; 1312.5344]

[Chester, Lee, Pufu, Yacoby; 1412.0334]

- CFTs with large central charge and large gap can have correlators computed from supergravity ($1/c_T$ expansion).

Basic playground

Superconformal field theories

↙ ↘

Supergravity

Chiral algebra or TQFT

[Maldacena; hep-th/9711200]

[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees; 1312.5344]

[Chester, Lee, Pufu, Yacoby; 1412.0334]

- CFTs with large central charge and large gap can have correlators computed from supergravity ($1/c_T$ expansion).
- CFTs with extended supersymmetry have nilpotent "$Q + S$" supercharge which singles out twisted configurations of correlators.
- Operators in cohomology generate a chiral algebra (6d, 4d) or topological QM (3d) but which one not always clear.

Basic playground

Superconformal field theories

↙ ↘

Supergravity

Chiral algebra or TQFT

[Maldacena; hep-th/9711200]

[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees; 1312.5344]

[Chester, Lee, Pufu, Yacoby; 1412.0334]

- CFTs with large central charge and large gap can have correlators computed from supergravity ($1/c_T$ expansion).
- CFTs with extended supersymmetry have nilpotent "$Q + S$" supercharge which singles out twisted configurations of correlators.
- Operators in cohomology generate a chiral algebra (6d, 4d) or topological QM (3d) but which one not always clear.
- Known $O(1/c_T)$ 4pt functions for maximal SUSY enable a detailed check [Alday, Zhou; 2006.06653] .

Basic playground

Superconformal field theories

↙ ↘

Supergravity $\longleftrightarrow$ Chiral algebra or TQFT

[Maldacena; hep-th/9711200]

[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees; 1312.5344]

[Chester, Lee, Pufu, Yacoby; 1412.0334]

- CFTs with large central charge and large gap can have correlators computed from supergravity ($1/c_T$ expansion).
- CFTs with extended supersymmetry have nilpotent "$Q + S$" supercharge which singles out twisted configurations of correlators.
- Operators in cohomology generate a chiral algebra (6d, 4d) or topological QM (3d) but which one not always clear.
- Known $O(1/c_T)$ 4pt functions for maximal SUSY enable a detailed check [Alday, Zhou; 2006.06653] .

Plan

1. Half-BPS 4pt functions
   - Superconformal kinematics
   - Holographic correlators in Mellin space
   - How to bootstrap them
2. Checks of chiral symmetry
   - $\mathcal{W}_N$ 4pt functions at tree level
   - Matching to 6d results
   - What changes in 4d
3. Exploring the topological sector
   - OPE coefficients from Mellin space
   - Finite crossing equations
   - Comparisons to matrix model results

Plan

1. Half-BPS 4pt functions
   - Superconformal kinematics
   - Holographic correlators in Mellin space
   - How to bootstrap them
2. Checks of chiral symmetry
   - $\mathcal{W}_N$ 4pt functions at tree level
   - Matching to 6d results
   - What changes in 4d
3. Exploring the topological sector
   - OPE coefficients from Mellin space
   - Finite crossing equations
   - Comparisons to matrix model results

Half-BPS operators dual to KK-modes

$AdS_7 \times S^4$	$AdS_5 \times S^5$	$AdS_4 \times S^7$

Half-BPS operators dual to KK-modes

$AdS_7 \times S^4$	$AdS_5 \times S^5$	$AdS_4 \times S^7$
$SO(6,2) \times SO(5)$ $\subset OSp(8^*\|4)$	$SO(4,2) \times SO(6)$ $\subset PSU(2,2\|4)$	$SO(3,2) \times SO(8)$ $\subset OSp(8\|4)$

Half-BPS operators dual to KK-modes

$AdS_7 \times S^4$	$AdS_5 \times S^5$	$AdS_4 \times S^7$
$SO(6,2) \times SO(5)$ $\subset OSp(8^*\|4)$	$SO(4,2) \times SO(6)$ $\subset PSU(2,2\|4)$	$SO(3,2) \times SO(8)$ $\subset OSp(8\|4)$
Rank-k symmetric tensors w/ $\ell = 0$, $\Delta = \epsilon k$ and $\epsilon = \frac{d-2}{2}$. $\mathcal{O}_k(x,t) = \mathcal{O}_k^{I_1 \dots I_k}(x) t_{I_1} \dots t_{I_k}$		

Half-BPS operators dual to KK-modes

$AdS_7 \times S^4$	$AdS_5 \times S^5$	$AdS_4 \times S^7$
$SO(6,2) \times SO(5)$ $\subset OSp(8^*\|4)$	$SO(4,2) \times SO(6)$ $\subset PSU(2,2\|4)$	$SO(3,2) \times SO(8)$ $\subset OSp(8\|4)$
Rank-k symmetric tensors w/ $\ell = 0$, $\Delta = \epsilon k$ and $\epsilon = \frac{d-2}{2}$. $\mathcal{O}_k(x,t) = \mathcal{O}_k^{I_1 \dots I_k}(x) t_{I_1} \dots t_{I_k}$		
$c_T \sim 280N^3$	$c_T \sim 30N^2$	$c_T \sim \frac{64\sqrt{2}}{3\pi} N^{\frac{3}{2}}$

Half-BPS operators dual to KK-modes

$AdS_7 \times S^4$	$AdS_5 \times S^5$	$AdS_4 \times S^7$
$SO(6,2) \times SO(5)$ $\subset OSp(8^*\|4)$	$SO(4,2) \times SO(6)$ $\subset PSU(2,2\|4)$	$SO(3,2) \times SO(8)$ $\subset OSp(8\|4)$
Rank-k symmetric tensors w/ $\ell = 0$, $\Delta = \epsilon k$ and $\epsilon = \frac{d-2}{2}$. $\mathcal{O}_k(x,t) = \mathcal{O}_k^{I_1 \dots I_k}(x) t_{I_1} \dots t_{I_k}$		
$c_T \sim 280N^3$	$c_T \sim 30N^2$	$c_T \sim \frac{64\sqrt{2}}{3\pi} N^{\frac{3}{2}}$
3pt C_{k_1,k_2,k_3} known in terms of $\beta_1 = \frac{1}{2}(k_3 + k_2 - k_1)$, etc. [Lee, Minwalla, Rangamani, Seiberg; hep-th/9806074] [Bastianelli, Zucchini; hep-th/9907047]		

Half-BPS operators dual to KK-modes

<table>
<tr><td>$AdS_7 \times S^4$</td><td>$AdS_5 \times S^5$</td><td>$AdS_4 \times S^7$</td></tr>
<tr><td>$SO(6,2) \times SO(5)$
$\subset OSp(8^*|4)$</td><td>$SO(4,2) \times SO(6)$
$\subset PSU(2,2|4)$</td><td>$SO(3,2) \times SO(8)$
$\subset OSp(8|4)$</td></tr>
<tr><td colspan="3">Rank-k symmetric tensors w/ $\ell = 0$, $\Delta = \epsilon k$ and $\epsilon = \frac{d-2}{2}$.
$\mathcal{O}_k(x,t) = \mathcal{O}_k^{I_1 \dots I_k}(x) t_{I_1} \dots t_{I_k}$</td></tr>
<tr><td>$c_T \sim 280 N^3$</td><td>$c_T \sim 30 N^2$</td><td>$c_T \sim \frac{64\sqrt{2}}{3\pi} N^{\frac{3}{2}}$</td></tr>
<tr><td colspan="3">3pt C_{k_1,k_2,k_3} known in terms of $\beta_1 = \frac{1}{2}(k_3 + k_2 - k_1)$, etc.
[Lee, Minwalla, Rangamani, Seiberg; hep-th/9806074] [Bastianelli, Zucchini; hep-th/9907047]</td></tr>
<tr><td>$\frac{2^{2\beta-2}\Gamma[\beta]}{(\pi N)^{\frac{3}{2}}} \prod_i \frac{\Gamma\left[\beta_i + \frac{1}{2}\right]}{\sqrt{\Gamma[2k_i - 1]}}$</td><td>$\frac{\sqrt{k_1 k_2 k_3}}{N}$</td><td>$\frac{\pi 2^{-\beta-\frac{1}{4}}}{N^{\frac{3}{4}}\Gamma\left[\frac{\beta+2}{2}\right]} \prod_i \frac{\sqrt{\Gamma[k_i+2]}}{\Gamma\left[\frac{\beta_i+1}{2}\right]}$</td></tr>
</table>

Four-point functions

Cross-ratios are built from $x_{ij} = x_i - x_j$ and $t_{ij} = t_i \cdot t_j$.

$$U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = \chi\chi', \quad V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-\chi)(1-\chi')$$

$$\sigma = \frac{t_{13} t_{24}}{t_{12} t_{34}} = \alpha\alpha', \quad \tau = \frac{t_{14} t_{23}}{t_{12} t_{34}} = (1-\alpha)(1-\alpha')$$

Four-point functions

Cross-ratios are built from $x_{ij} = x_i - x_j$ and $t_{ij} = t_i \cdot t_j$.

$$U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = \chi\chi', \quad V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-\chi)(1-\chi')$$

$$\sigma = \frac{t_{13} t_{24}}{t_{12} t_{34}} = \alpha\alpha', \quad \tau = \frac{t_{14} t_{23}}{t_{12} t_{34}} = (1-\alpha)(1-\alpha')$$

Overload notation for conformal and R symmetry in 1d, 2d.

$$\chi = \frac{z_{12} z_{34}}{z_{13} z_{24}}, \quad \alpha = \frac{y_{12} y_{34}}{y_{13} y_{24}}$$

Four-point functions

Cross-ratios are built from $x_{ij} = x_i - x_j$ and $t_{ij} = t_i \cdot t_j$.

$$U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = \chi\chi', \quad V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-\chi)(1-\chi')$$
$$\sigma = \frac{t_{13} t_{24}}{t_{12} t_{34}} = \alpha\alpha', \quad \tau = \frac{t_{14} t_{23}}{t_{12} t_{34}} = (1-\alpha)(1-\alpha')$$

Overload notation for conformal and R symmetry in 1d, 2d.

$$\chi = \frac{z_{12} z_{34}}{z_{13} z_{24}}, \quad \alpha = \frac{y_{12} y_{34}}{y_{13} y_{24}}$$

There are two common conventions.

$$\langle \mathcal{O}_1(z_1)\mathcal{O}_2(z_2)\mathcal{O}_3(z_3)\mathcal{O}_4(z_4)\rangle =$$
$$\left(\frac{z_{24}}{z_{14}}\right)^{h_{12}} \left(\frac{z_{14}}{z_{13}}\right)^{h_{34}} \frac{\mathcal{F}(\chi)}{z_{12}^{h_1+h_2} z_{34}^{h_3+h_4}}$$

Four-point functions

Cross-ratios are built from $x_{ij} = x_i - x_j$ and $t_{ij} = t_i \cdot t_j$.

$$U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = \chi\chi', \quad V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-\chi)(1-\chi')$$
$$\sigma = \frac{t_{13} t_{24}}{t_{12} t_{34}} = \alpha\alpha', \quad \tau = \frac{t_{14} t_{23}}{t_{12} t_{34}} = (1-\alpha)(1-\alpha')$$

Overload notation for conformal and R symmetry in 1d, 2d.

$$\chi = \frac{z_{12} z_{34}}{z_{13} z_{24}}, \quad \alpha = \frac{y_{12} y_{34}}{y_{13} y_{24}}$$

There are two common conventions.

$$\langle \mathcal{O}_1(z_1, y_1) \mathcal{O}_2(z_2, y_2) \mathcal{O}_3(z_3, y_3) \mathcal{O}_4(z_4, y_4) \rangle =$$
$$\left(\frac{z_{24}}{z_{14}} \right)^{h_{12}} \left(\frac{z_{14}}{z_{13}} \right)^{h_{34}} \left(\frac{y_{14}}{y_{24}} \right)^{j_{12}} \left(\frac{y_{13}}{y_{14}} \right)^{j_{34}} \frac{y_{12}^{j_1+j_2} y_{34}^{j_3+j_4}}{z_{12}^{h_1+h_2} z_{34}^{h_3+h_4}} \mathcal{F}(\chi; \alpha)$$

Four-point functions

Define extremality using i-th smallest weight $k_{\bar{\imath}}$.

$$\mathcal{E} = \begin{cases} \frac{k_{\bar{1}}+k_{\bar{2}}+k_{\bar{3}}-k_{\bar{4}}}{2}, & k_{\bar{1}}+k_{\bar{4}} \geq k_{\bar{2}}+k_{\bar{3}} \\ k_{\bar{1}}, & k_{\bar{1}}+k_{\bar{4}} < k_{\bar{2}}+k_{\bar{3}} \end{cases}$$

Four-point functions

Define extremality using i-th smallest weight $k_{\bar{\imath}}$.

$$\mathcal{E} = \begin{cases} \frac{k_{\bar{1}}+k_{\bar{2}}+k_{\bar{3}}-k_{\bar{4}}}{2}, & k_{\bar{1}} + k_{\bar{4}} \geq k_{\bar{2}} + k_{\bar{3}} \\ k_{\bar{1}}, & k_{\bar{1}} + k_{\bar{4}} < k_{\bar{2}} + k_{\bar{3}} \end{cases}$$

Polynomial-friendly prefactor is

$$t_{\overline{34}}^{\frac{1}{2}(k_{\bar{3}}+k_{\bar{4}}-k_{\bar{1}}-k_{\bar{2}})} t_{\overline{24}}^{\frac{1}{2}(k_{\bar{2}}+k_{\bar{4}}-k_{\bar{1}}-k_{\bar{3}})} t_{\overline{23}}^{\frac{1}{2}(k_{\bar{1}}+k_{\bar{2}}+k_{\bar{3}}-k_{\bar{4}})-\mathcal{E}} t_{\overline{14}}^{k_{\bar{1}}-\mathcal{E}} (t_{\overline{12}} t_{\overline{34}})^{\mathcal{E}}.$$

Four-point functions

Define extremality using i-th smallest weight $k_{\bar{\imath}}$.

$$\mathcal{E} = \begin{cases} \frac{k_{\bar{1}}+k_{\bar{2}}+k_{\bar{3}}-k_{\bar{4}}}{2}, & k_{\bar{1}} + k_{\bar{4}} \geq k_{\bar{2}} + k_{\bar{3}} \\ k_{\bar{1}}, & k_{\bar{1}} + k_{\bar{4}} < k_{\bar{2}} + k_{\bar{3}} \end{cases}$$

Polynomial-friendly prefactor is

$$\left[\frac{t_{34}}{x_{34}^{2\epsilon}}\right]^{\frac{1}{2}(k_3+k_4-k_1-k_2)} \left[\frac{t_{24}}{x_{24}^{2\epsilon}}\right]^{\frac{1}{2}(k_2+k_3-k_1-k_3)} \left[\frac{t_{23}}{x_{23}^{2\epsilon}}\right]^{\frac{1}{2}(k_1+k_2+k_3-k_4)-\mathcal{E}} \left[\frac{t_{14}}{x_{14}^{2\epsilon}}\right]^{k_1-\mathcal{E}} \left[\frac{t_{12}t_{34}}{x_{12}^{2\epsilon}x_{34}^{2\epsilon}}\right]^{\mathcal{E}}.$$

Use this for $k_1 \leq k_2 \leq k_3 \leq k_4$, trivial permutation otherwise.

Four-point functions

Define extremality using i-th smallest weight $k_{\bar{\imath}}$.

$$\mathcal{E} = \begin{cases} \frac{k_{\bar{1}}+k_{\bar{2}}+k_{\bar{3}}-k_{\bar{4}}}{2}, & k_{\bar{1}} + k_{\bar{4}} \geq k_{\bar{2}} + k_{\bar{3}} \\ k_{\bar{1}}, & k_{\bar{1}} + k_{\bar{4}} < k_{\bar{2}} + k_{\bar{3}} \end{cases}$$

Polynomial-friendly prefactor is

$$\left[\frac{t_{34}}{x_{34}^{2\epsilon}}\right]^{\frac{1}{2}(k_3+k_4-k_1-k_2)} \left[\frac{t_{24}}{x_{24}^{2\epsilon}}\right]^{\frac{1}{2}(k_2+k_3-k_1-k_3)} \left[\frac{t_{23}}{x_{23}^{2\epsilon}}\right]^{\frac{1}{2}(k_1+k_2+k_3-k_4)-\mathcal{E}} \left[\frac{t_{14}}{x_{14}^{2\epsilon}}\right]^{k_1-\mathcal{E}} \left[\frac{t_{12}t_{34}}{x_{12}^{2\epsilon}x_{34}^{2\epsilon}}\right]^{\mathcal{E}}.$$

Use this for $k_1 \leq k_2 \leq k_3 \leq k_4$, trivial permutation otherwise. Dynamical $G(U, V; \sigma, \tau)$ satisfies superconformal Ward identity.

$$\left[\chi'\partial_{\chi'} - \epsilon\alpha'\partial_{\alpha'}\right] G(\chi, \chi'; \alpha, \alpha')\Big|_{\alpha'=1/\chi'} = 0$$

Holographic four-point functions

Necessary Witten diagram calculations in position space are possible but tedious [D'Hoker, Freedman, Mathur, Matusis, Rastelli; hep-th/9903196] .

Holographic four-point functions

Necessary Witten diagram calculations in position space are possible but tedious [D'Hoker, Freedman, Mathur, Matusis, Rastelli; hep-th/9903196] .
Much better to use Mellin space [Mack; 0907.2407] .

$$G(U,V) = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2} - \frac{\epsilon}{2}(k_1+k_2-\mathcal{E})} V^{\frac{t}{2} + \frac{\epsilon}{2}(k_1-k4-\mathcal{E})} \mathcal{M}(s,t)$$
$$\Gamma\left[\frac{\Delta_1+\Delta_2-s}{2}\right]\Gamma\left[\frac{\Delta_1+\Delta_4-t}{2}\right]\Gamma\left[\frac{\Delta_1+\Delta_3-u}{2}\right]$$
$$\Gamma\left[\frac{\Delta_3+\Delta_4-s}{2}\right]\Gamma\left[\frac{\Delta_2+\Delta_3-t}{2}\right]\Gamma\left[\frac{\Delta_2+\Delta_4-u}{2}\right]$$

Holographic four-point functions

Necessary Witten diagram calculations in position space are possible but tedious [D'Hoker, Freedman, Mathur, Matusis, Rastelli; hep-th/9903196] . Much better to use Mellin space [Mack; 0907.2407] .

$$G(U,V) = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2}-\frac{\epsilon}{2}(k_1+k_2-\mathcal{E})} V^{\frac{t}{2}+\frac{\epsilon}{2}(k_1-k4-\mathcal{E})} \mathcal{M}(s,t)$$
$$\Gamma\left[\frac{\Delta_1+\Delta_2-s}{2}\right]\Gamma\left[\frac{\Delta_1+\Delta_4-t}{2}\right]\Gamma\left[\frac{\Delta_1+\Delta_3-u}{2}\right]$$
$$\Gamma\left[\frac{\Delta_3+\Delta_4-s}{2}\right]\Gamma\left[\frac{\Delta_2+\Delta_3-t}{2}\right]\Gamma\left[\frac{\Delta_2+\Delta_4-u}{2}\right]$$

Conformal blocks and Witten diagrams (for $\tau = \Delta - \ell$) both become

$$\mathcal{M}_{\tau,\ell}(s,t) = \sum_{m=0}^{\infty} \frac{Q^{\tau}_{\ell,m}(t)}{m!\Gamma\left[\frac{\Delta_1+\Delta_2-\tau}{2}-m\right]\Gamma\left[\frac{\Delta_3+\Delta_4-\tau}{2}-m\right](s-\tau-2m)}.$$

Holographic four-point functions

Necessary Witten diagram calculations in position space are possible but tedious [D'Hoker, Freedman, Mathur, Matusis, Rastelli; hep-th/9903196] . Much better to use Mellin space [Mack; 0907.2407] .

$$G(U,V) = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2} - \frac{\epsilon}{2}(k_1+k_2-\mathcal{E})} V^{\frac{t}{2} + \frac{\epsilon}{2}(k_1-k4-\mathcal{E})} \mathcal{M}(s,t)$$
$$\Gamma\left[\frac{\Delta_1+\Delta_2-s}{2}\right]\Gamma\left[\frac{\Delta_1+\Delta_4-t}{2}\right]\Gamma\left[\frac{\Delta_1+\Delta_3-u}{2}\right]$$
$$\Gamma\left[\frac{\Delta_3+\Delta_4-s}{2}\right]\Gamma\left[\frac{\Delta_2+\Delta_3-t}{2}\right]\Gamma\left[\frac{\Delta_2+\Delta_4-u}{2}\right]$$

Conformal blocks and Witten diagrams (for $\tau = \Delta - \ell$) both become

$$\mathcal{M}_{\tau,\ell}(s,t) = \sum_{m=0}^{\infty} \frac{Q^{\tau}_{\ell,m}(t)}{m!\Gamma\left[\frac{\Delta_1+\Delta_2-\tau}{2} - m\right]\Gamma\left[\frac{\Delta_3+\Delta_4-\tau}{2} - m\right](s-\tau-2m)}.$$

Hardest part is solving for contact terms [Arutyunov, Frolov; hep-th/9912210] .

Holographic four-point functions

Necessary Witten diagram calculations in position space are possible but tedious [D'Hoker, Freedman, Mathur, Matusis, Rastelli; hep-th/9903196] .
Much better to use Mellin space [Mack; 0907.2407] .

$$G(U,V) = \int_{-i\infty}^{i\infty} \frac{ds dt}{(4\pi i)^2} U^{\frac{s}{2} - \frac{\epsilon}{2}(k_1 + k_2 - \mathcal{E})} V^{\frac{t}{2} + \frac{\epsilon}{2}(k_1 - k4 - \mathcal{E})} \mathcal{M}(s,t)$$
$$\Gamma\left[\frac{\Delta_1 + \Delta_2 - s}{2}\right] \Gamma\left[\frac{\Delta_1 + \Delta_4 - t}{2}\right] \Gamma\left[\frac{\Delta_1 + \Delta_3 - u}{2}\right]$$
$$\Gamma\left[\frac{\Delta_3 + \Delta_4 - s}{2}\right] \Gamma\left[\frac{\Delta_2 + \Delta_3 - t}{2}\right] \Gamma\left[\frac{\Delta_2 + \Delta_4 - u}{2}\right]$$

Conformal blocks and Witten diagrams (for $\tau = \Delta - \ell$) both become

$$\mathcal{M}_{\tau,\ell}(s,t) = \sum_{m=0}^{\infty} \frac{Q^{\tau}_{\ell,m}(t,u)}{m!\Gamma\left[\frac{\Delta_1+\Delta_2-\tau}{2} - m\right]\Gamma\left[\frac{\Delta_3+\Delta_4-\tau}{2} - m\right](s - \tau - 2m)}.$$

Hardest part is solving for contact terms [Arutyunov, Frolov; hep-th/9912210] .

Modern methods

Bootstrap techniques can help complete the ansatz.

$$\mathcal{M}(s,t) = \sum_{\mathcal{O}} C_{12\mathcal{O}} C_{34\mathcal{O}} \mathcal{M}_{\mathcal{O}}(s,t) + C_{14\mathcal{O}} C_{23\mathcal{O}} \mathcal{M}_{\mathcal{O}}(t,s) + C_{13\mathcal{O}} C_{24\mathcal{O}} \mathcal{M}_{\mathcal{O}}(u,t)$$
$$+ w_0 + w_1 s + w_2 t$$

Modern methods

Bootstrap techniques can help complete the ansatz.

$$\mathcal{M}(s,t) = \sum_{\mathcal{O}} C_{12\mathcal{O}} C_{34\mathcal{O}} \mathcal{M}_{\mathcal{O}}(s,t) + C_{14\mathcal{O}} C_{23\mathcal{O}} \mathcal{M}_{\mathcal{O}}(t,s) + C_{13\mathcal{O}} C_{24\mathcal{O}} \mathcal{M}_{\mathcal{O}}(u,t)$$
$$+ w_0 + w_1 s + w_2 t$$

1. Impose Mellin superconformal Ward identity [Zhou; 1712.02800] .

Modern methods

Bootstrap techniques can help complete the ansatz.

$$\mathcal{M}(s,t) = \sum_{\mathcal{O}} C_{12\mathcal{O}} C_{34\mathcal{O}} \mathcal{M}_{\mathcal{O}}(s,t) + C_{14\mathcal{O}} C_{23\mathcal{O}} \mathcal{M}_{\mathcal{O}}(t,s) + C_{13\mathcal{O}} C_{24\mathcal{O}} \mathcal{M}_{\mathcal{O}}(u,t)$$
$$+ w_0 + w_1 s + w_2 t$$

1. Impose Mellin superconformal Ward identity [Zhou; 1712.02800] .
2. Symmetrize at the level of supermultiplets [Alday, Zhou; 2006.06653] .

$$\mathcal{R}^{ij}_{\tau,m}(t) \propto t^2 + q_1(m)t + q_2(m)$$
$$s + t + u = \epsilon \Sigma_k, \quad s = \tau + 2m$$

Modern methods

Bootstrap techniques can help complete the ansatz.

$$\mathcal{M}(s,t) = \sum_{\mathcal{O}} C_{12\mathcal{O}} C_{34\mathcal{O}} \mathcal{M}_{\mathcal{O}}(s,t) + C_{14\mathcal{O}} C_{23\mathcal{O}} \mathcal{M}_{\mathcal{O}}(t,s) + C_{13\mathcal{O}} C_{24\mathcal{O}} \mathcal{M}_{\mathcal{O}}(u,t)$$
$$+ w_0 + w_1 s + w_2 t$$

1. Impose Mellin superconformal Ward identity [Zhou; 1712.02800] .
2. Symmetrize at the level of supermultiplets [Alday, Zhou; 2006.06653] .

$$\mathcal{R}^{ij}_{\tau,m}(t) \propto t^2 + q_1(m)t + q_2(m)$$
$$s + t + u = \epsilon \Sigma_k, \quad s = \tau + 2m$$

Produces $AdS_{d+1} \times S^{\mathrm{d}-1}$ solution with no contact terms.

$$\mathcal{M}(s,t;\sigma,\tau) = \mathcal{M}^{(s)}(s,t;\sigma,\tau) + \mathcal{M}^{(t)}(s,t;\sigma,\tau) + \mathcal{M}^{(u)}(s,t;\sigma,\tau)$$
$$\mathcal{M}^{(s)}(s,t;\sigma,\tau) = \sum_p \sum_{m=0}^{\infty} \sum_{0 \leq i+j \leq \mathcal{E}} \sigma^i \tau^j \frac{\mathcal{R}^{ij}_{p,m}(t,u)}{s - \epsilon p - 2m}$$

Modern methods

$$\mathcal{R}^{ij}_{p,m}(t,u) = \frac{C_{k_1k_2p}C_{k_3k_4p}K^{ij}_p(t,u)H^{ij}_{p,m}}{i!j!m!\Gamma\left[\frac{\epsilon}{2}(k_1+k_2-p)-m\right]\Gamma\left[\frac{\epsilon}{2}(k_3+k_4-p)-m\right]}$$

$$\begin{aligned}K^{ij}_{p,m} &= 2i(2i+\kappa_u)t^-t^+ - 2i(\mathtt{d}-6+\kappa_t)u^+t^- + (u,i\leftrightarrow t,j)\\ &+\frac{1}{4}(2p-\kappa_t-\kappa_u)(2p++\kappa_t+\kappa_u+2\mathtt{d}-12)(2\epsilon j-t^-)(2\epsilon i-u^-)\\ &+4\epsilon ij[t^+(\mathtt{d}-6+\kappa_u)+u^+(\mathtt{d}-6+\kappa_t)] - 8ijt^+u^+\end{aligned}$$

$$t^\pm = t\pm\frac{\epsilon}{2}\kappa_t - \frac{\epsilon}{2}\Sigma_k, \quad u^\pm = u\pm\frac{\epsilon}{2}\kappa_u - \frac{\epsilon}{2}\Sigma_k$$

$$\kappa_s = |k_1+k_2-k_3-k_4|, \quad \kappa_t = |k_1+k_4-k_2-k_3|, \quad \kappa_u = |k_2+k_4-k_1-k_3|$$

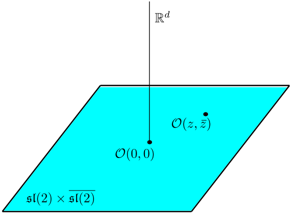
$$H^{ij}_{p,m} = \frac{2^{-\frac{1}{3}(2\epsilon-\mathtt{d}+13)}}{[m+1+\epsilon(p-1)]!}\frac{(-1)^{i+j+\frac{2p-\kappa_t-\kappa_u}{4}}}{\left[i+\frac{\kappa_u}{2}\right]!\left[j+\frac{\kappa_t}{2}\right]!}\frac{\Gamma\left[\frac{2p+2\mathtt{d}-12+\Sigma_k-\kappa_s-4(\mathcal{E}-i-j)}{4}\right]}{\Gamma\left[\frac{2p+4-\Sigma_k+\kappa_s+4(\mathcal{E}-i-j)}{4}\right]}$$

$$\times\prod_x\frac{\Gamma[x]}{\Gamma[\epsilon x]}\prod_y\frac{\Gamma[\epsilon y]}{\Gamma[y]}, \quad x\in\left\{\frac{p\pm k_{12}}{2},\frac{p\pm k_{34}}{2}\right\}, \quad y\in\left\{p,p+\frac{\mathtt{d}-6}{2}\right\}$$

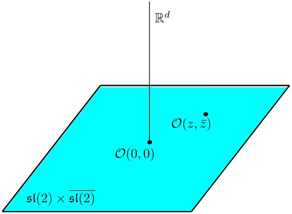
Plan

1. Half-BPS 4pt functions
   - Superconformal kinematics
   - Holographic correlators in Mellin space
   - How to bootstrap them
2. Checks of chiral symmetry
   - $\mathcal{W}_N$ 4pt functions at tree level
   - Matching to 6d results
   - What changes in 4d
3. Exploring the topological sector
   - OPE coefficients from Mellin space
   - Finite crossing equations
   - Comparisons to matrix model results

Chiral algebra / SCFT correspondence

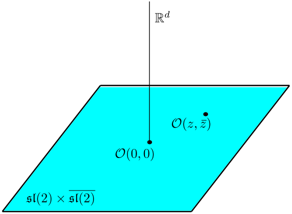


Chiral algebra / SCFT correspondence



Consider $\mathfrak{sl}(2) \times \mathfrak{sl}(2|2)$ preserving plane.

Chiral algebra / SCFT correspondence

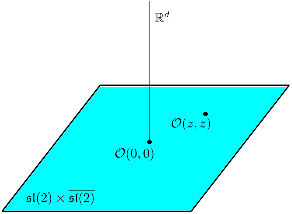


Consider $\mathfrak{sl}(2) \times \mathfrak{sl}(2|2)$ preserving plane.

Fermionic generators give nilpotent $\mathbb{Q}$.

R-sym generators give exact $\bar{L}_{0,\pm 1} - R_{0,\pm 1}$.

Chiral algebra / SCFT correspondence



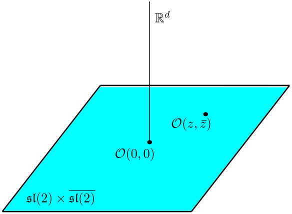
Consider $\mathfrak{sl}(2) \times \mathfrak{sl}(2|2)$ preserving plane.

Fermionic generators give nilpotent $\mathbb{Q}$.

R-sym generators give exact $\bar{L}_{0,\pm 1} - R_{0,\pm 1}$.

Twisted translation predicted by superconformal Ward identity.

$$[\mathcal{O}(z)] = \mathcal{O}^{b_1 \dots b_{2j}}(z,\bar{z}) u_{b_1}(\bar{z}) \dots u_{b_{2j}}(\bar{z}), \quad u(\bar{z}) = (1,\bar{z})$$



Consider $\mathfrak{sl}(2)\times\mathfrak{sl}(2|2)$ preserving plane.

Fermionic generators give nilpotent $\mathbb{Q}$.

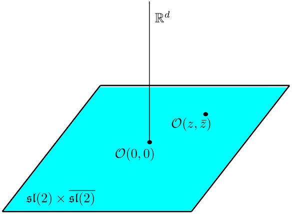
R-sym generators give exact $\bar{L}_{0,\pm 1} - R_{0,\pm 1}$.

Twisted translation predicted by superconformal Ward identity.

$$[\mathcal{O}(z)] = \mathcal{O}^{b_1\dots b_{2j}}(z,\bar{z})u_{b_1}(\bar{z})\dots u_{b_{2j}}(\bar{z}), \quad u(\bar{z}) = (1,\bar{z})$$

This works for 4d $\mathcal{N}=2,3,4$ [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees; 1312.5344] and 6d $\mathcal{N}=(2,0)$ [Beem, Rastelli, van Rees; 1404.1079] .

Chiral algebra / SCFT correspondence



Consider $\mathfrak{sl}(2) \times \mathfrak{sl}(2|2)$ preserving plane.

Fermionic generators give nilpotent $\mathbb{Q}$.

R-sym generators give exact $\bar{L}_{0,\pm 1} - R_{0,\pm 1}$.

Twisted translation predicted by superconformal Ward identity.

$$[\mathcal{O}(z)] = \mathcal{O}^{b_1 \dots b_{2j}}(z,\bar{z}) u_{b_1}(\bar{z}) \dots u_{b_{2j}}(\bar{z}), \quad u(\bar{z}) = (1,\bar{z})$$

This works for 4d $\mathcal{N} = 2, 3, 4$ [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees; 1312.5344] and 6d $\mathcal{N} = (2,0)$ [Beem, Rastelli, van Rees; 1404.1079] .

$$\begin{aligned} T &\implies \textit{strong generator} \\ \Lambda = (TT) - \frac{3}{10}\partial^2 T &\implies \textit{generator} \\ \partial T, \partial^2 T, \partial(TT), \dots &\implies \textit{everything else} \end{aligned}$$

$\mathcal{W}$-algebras

Consider $\mathcal{W}_3$ algebra [Zamolodchikov; 1985] .

$$[W_m, W_n] \supset \delta_{m+n,0}, L_{m+n}, \sum_p (L_{m+n-p} L_p)$$

$\mathcal{W}$-algebras

Consider $\mathcal{W}_3$ algebra [Zamolodchikov; 1985] .

$$W(z)W(0) \sim \frac{c/3}{z^6} + \frac{c_1 T(0)}{z^4} + \frac{c_2 \partial T(0)}{z^3} + \frac{1}{z^2}[c_3 \Lambda(0) + c_4 \partial^2 T(0)] + \frac{1}{z}[c_5 \partial \Lambda(0) + c_6 \partial^3 T]$$

$\mathcal{W}$-algebras

Consider $\mathcal{W}_3$ algebra [Zamolodchikov; 1985] . Associativity fixes $\gamma = \frac{16}{22+5c}$.

$$W(z)W(0) \sim \frac{c/3}{z^6} + \frac{2T(0)}{z^4} + \frac{\partial T(0)}{z^3} + \frac{1}{z^2}\left[2\gamma\Lambda(0) + \frac{3}{10}\partial^2 T(0)\right] + \frac{1}{z}\left[\gamma\partial\Lambda(0) + \frac{1}{15}\partial^3 T\right]$$

$\mathcal{W}$-algebras

Consider $\mathcal{W}_3$ algebra [Zamolodchikov; 1985] . Associativity fixes $\gamma = \frac{16}{22+5c}$.

$$
\begin{aligned}
W(z)W(0) \sim \frac{c/3}{z^6} + \frac{2T(0)}{z^4} + \frac{\partial T(0)}{z^3} \\
+ \frac{1}{z^2}\left[2\gamma\Lambda(0) + \frac{3}{10}\partial^2 T(0)\right] + \frac{1}{z}\left[\gamma\partial\Lambda(0) + \frac{1}{15}\partial^3 T\right]
\end{aligned}
$$

Also have $\mathcal{W}_N = \mathcal{W}[\mathfrak{sl}(N)]$ for spins $2, 3, \ldots, N$.

$\mathcal{W}$-algebras

Consider $\mathcal{W}_3$ algebra [Zamolodchikov; 1985] . Associativity fixes $\gamma = \frac{16}{22+5c}$.

$$W(z)W(0) \sim \frac{c/3}{z^6} + \frac{2T(0)}{z^4} + \frac{\partial T(0)}{z^3} + \frac{1}{z^2}\left[2\gamma\Lambda(0) + \frac{3}{10}\partial^2 T(0)\right] + \frac{1}{z}\left[\gamma\partial\Lambda(0) + \frac{1}{15}\partial^3 T\right]$$

Also have $\mathcal{W}_N = \mathcal{W}[\mathfrak{sl}(N)]$ for spins $2, 3, \dots, N$.

$$\mathcal{O}_{h_1}(z)\mathcal{O}_{h_2}(0) = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}} \sum_{m=0}^{\infty} \frac{(h_{12}+h)_m}{m!(2h)_m} \frac{\partial^m \mathcal{O}_h(0)}{z^{h_1+h_2-h-m}}$$

$\mathcal{W}$-algebras

Consider $\mathcal{W}_3$ algebra [Zamolodchikov; 1985] . Associativity fixes $\gamma = \frac{16}{22+5c}$.

$$W(z)W(0) \sim \frac{c/3}{z^6} + \frac{2T(0)}{z^4} + \frac{\partial T(0)}{z^3} + \frac{1}{z^2}\left[2\gamma\Lambda(0) + \frac{3}{10}\partial^2 T(0)\right] + \frac{1}{z}\left[\gamma\partial\Lambda(0) + \frac{1}{15}\partial^3 T\right]$$

Also have $\mathcal{W}_N = \mathcal{W}[\mathfrak{sl}(N)]$ for spins $2, 3, \dots, N$.

$$\mathcal{O}_{h_1}(z)\mathcal{O}_{h_2}(0) = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}} \sum_{m=0}^{\infty} \frac{(h_{12}+h)_m}{m!(2h)_m} \frac{\partial^m \mathcal{O}_h(0)}{z^{h_1+h_2-h-m}}$$

Leads to easy and hard expansion of 4pt functions.

$$\mathcal{F}_{1234}(\chi) = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}} g_h^{h_{12},h_{34}}(\chi)$$

$\mathcal{W}$-algebras

Consider $\mathcal{W}_3$ algebra [Zamolodchikov; 1985] . Associativity fixes $\gamma = \frac{16}{22+5c}$.

$$W(z)W(0) \sim \frac{c/3}{z^6} + \frac{2T(0)}{z^4} + \frac{\partial T(0)}{z^3} + \frac{1}{z^2}\left[2\gamma\Lambda(0) + \frac{3}{10}\partial^2 T(0)\right] + \frac{1}{z}\left[\gamma\partial\Lambda(0) + \frac{1}{15}\partial^3 T\right]$$

Also have $\mathcal{W}_N = \mathcal{W}[\mathfrak{sl}(N)]$ for spins $2, 3, \dots, N$.

$$\mathcal{O}_{h_1}(z)\mathcal{O}_{h_2}(0) = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}} \sum_{m=0}^{\infty} \frac{(h_{12}+h)_m}{m!(2h)_m} \frac{\partial^m \mathcal{O}_h(0)}{z^{h_1+h_2-h-m}}$$

Leads to easy and hard expansion of 4pt functions.

$$\begin{aligned} \mathcal{F}_{1234}(\chi) &= \sum_{\mathcal{O}} \lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}} g_h^{h_{12},h_{34}}(\chi) \\ &= \sum_{\mathcal{O}} \lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}} g_h^{h_{12},h_{34}}(\chi)\Big|_{sing} \\ &+ \sum_{\mathcal{O}} \lambda_{14\mathcal{O}}\lambda_{23\mathcal{O}} \frac{\chi^{h_1+h_2}}{(1-\chi)^{h_2+h_3}} g_h^{h_{32},h_{14}}(1-\chi)\Big|_{sing} \end{aligned}$$

$\mathcal{W}_N$ for large N

6d $\mathcal{N} = (2, 0)$ chiral algebra is $\mathcal{W}_N$ with $c = 4N^3 - 3N - 1$?

$$W^{(k_1)}(z)W^{(k_2)}(0) = \frac{\delta_{12}}{z^{2k_1}} + \sum_{p=|k_{12}|+2}^{k_1+k_2-2} C_{k_1 k_2 p} \sum_{m=0}^{k_1+k_2-p-1} \frac{(k_{12}+p)_m}{m!(2p)_m} \frac{\partial^m W^{(p)}(0)}{z^{k_1+k_2-p-m}} + \dots$$

$\mathcal{W}_N$ for large N

6d $\mathcal{N} = (2,0)$ chiral algebra is $\mathcal{W}_N$ with $c = 4N^3 - 3N - 1$?

$$W^{(k_1)}(z)W^{(k_2)}(0) = \frac{\delta_{12}}{z^{2k_1}} + \sum_{p=|k_{12}|+2}^{k_1+k_2-2} C_{k_1 k_2 p} \sum_{m=0}^{k_1+k_2-p-1} \frac{(k_{12}+p)_m}{m!(2p)_m} \frac{\partial^m W^{(p)}(0)}{z^{k_1+k_2-p-m}} + \dots$$

Gives crossing symmetric 4pt function for $k_2 \leq k_1 \leq k_3 \leq k_4$.

$$\frac{\mathcal{F}_{1234}(\chi)}{\chi^{k_1+k_2}} = \sum_{p=max(|k_{12}|,|k_{34}|)+2}^{k_1+k_2-2} C_{k_1 k_2 p} C_{k_3 k_4 p} \sum_{m=0}^{k_1+k_2-p-1} \frac{(p-k_{12})_m (p+k_{34})_m}{m!(2p)_m \chi^{k_1+k_2-p-m}} + (s \leftrightarrow t)$$

$\mathcal{W}_N$ for large N

6d $\mathcal{N} = (2,0)$ chiral algebra is $\mathcal{W}_N$ with $c = 4N^3 - 3N - 1$?

$$W^{(k_1)}(z)W^{(k_2)}(0) = \frac{\delta_{12}}{z^{2k_1}} + \sum_{p=|k_{12}|+2}^{k_1+k_2-2} C_{k_1 k_2 p} \sum_{m=0}^{k_1+k_2-p-1} \frac{(k_{12}+p)_m}{m!(2p)_m} \frac{\partial^m W^{(p)}(0)}{z^{k_1+k_2-p-m}} + \dots$$

Gives crossing symmetric 4pt function for $k_2 \leq k_1 \leq k_3 \leq k_4$.

$$\frac{\mathcal{F}_{1234}(\chi)}{\chi^{k_1+k_2}} = \sum_{p=max(|k_{12}|,|k_{34}|)+2}^{k_1+k_2-2} C_{k_1 k_2 p} C_{k_3 k_4 p} \sum_{m=0}^{k_1+k_2-p-1} \frac{(p-k_{12})_m (p+k_{34})_m}{m!(2p)_m \chi^{k_1+k_2-p-m}} + (s \leftrightarrow t)$$

Should agree with 6d correlator under $\alpha = \alpha' = \frac{1}{\chi'}$.

$\mathcal{W}_N$ for large N

6d $\mathcal{N} = (2,0)$ chiral algebra is $\mathcal{W}_N$ with $c = 4N^3 - 3N - 1$?

$$W^{(k_1)}(z)W^{(k_2)}(0) = \frac{\delta_{12}}{z^{2k_1}} + \sum_{p=|k_{12}|+2}^{k_1+k_2-2} C_{k_1k_2p} \sum_{m=0}^{k_1+k_2-p-1} \frac{(k_{12}+p)_m}{m!(2p)_m} \frac{\partial^m W^{(p)}(0)}{z^{k_1+k_2-p-m}} + \dots$$

Gives crossing symmetric 4pt function for $k_2 \leq k_1 \leq k_3 \leq k_4$.

$$\frac{\mathcal{F}_{1234}(\chi)}{\chi^{k_1+k_2}} = \sum_{p=max(|k_{12}|,|k_{34}|)+2}^{k_1+k_2-2} C_{k_1k_2p}C_{k_3k_4p} \sum_{m=0}^{k_1+k_2-p-1} \frac{(p-k_{12})_m(p+k_{34})_m}{m!(2p)_m\chi^{k_1+k_2-p-m}} + (s \leftrightarrow t)$$

Should agree with 6d correlator under $\alpha = \alpha' = \frac{1}{\chi'}$.

$$\frac{\mathcal{G}\left(\frac{U}{V}, \frac{1}{V}; \tau, \sigma\right)}{(\sigma U^2)^{\mathcal{E}-\frac{1}{2}(k_1+k_2)}} = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2}} V^{k_1+k_4-\frac{s+t}{2}} \sigma^{\frac{1}{2}(k_1+k_2)-\mathcal{E}} \mathcal{M}(s,t;\tau,\sigma)\Gamma_{\{k_i\}}$$

$\mathcal{W}_N$ for large N

6d $\mathcal{N} = (2,0)$ chiral algebra is $\mathcal{W}_N$ with $c = 4N^3 - 3N - 1$?

$$W^{(k_1)}(z)W^{(k_2)}(0) = \frac{\delta_{12}}{z^{2k_1}} + \sum_{p=|k_{12}|+2}^{k_1+k_2-2} C_{k_1k_2p} \sum_{m=0}^{k_1+k_2-p-1} \frac{(k_{12}+p)_m}{m!(2p)_m} \frac{\partial^m W^{(p)}(0)}{z^{k_1+k_2-p-m}} + \dots$$

Gives crossing symmetric 4pt function for $k_2 \leq k_1 \leq k_3 \leq k_4$.

$$\frac{\mathcal{F}_{1234}(\chi)}{\chi^{k_1+k_2}} = \sum_{p=max(|k_{12}|,|k_{34}|)+2}^{k_1+k_2-2} C_{k_1k_2p}C_{k_3k_4p} \sum_{m=0}^{k_1+k_2-p-1} \frac{(p-k_{12})_m(p+k_{34})_m}{m!(2p)_m\chi^{k_1+k_2-p-m}} + (s \leftrightarrow t)$$

Should agree with 6d correlator under $\alpha = \alpha' = \frac{1}{\chi'}$.

$$\frac{\mathcal{G}\left(\frac{U}{V}, \frac{1}{V}; \tau, \sigma\right)}{(\sigma U^2)^{\mathcal{E}-\frac{1}{2}(k_1+k_2)}} = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2}} V^{k_1+k_4-\frac{s+t}{2}} \sigma^{\frac{1}{2}(k_1+k_2)-\mathcal{E}} \mathcal{M}(s,t;\tau,\sigma)\Gamma_{\{k_i\}}$$

$$\propto \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} (1-\chi')^{\frac{u}{2}-k_2-k_3} \sum_{0\leq i+j\leq\mathcal{E}} \sigma^i(1-\chi')^{2j} \dots$$

$\mathcal{W}_N$ for large N

6d $\mathcal{N} = (2,0)$ chiral algebra is $\mathcal{W}_N$ with $c = 4N^3 - 3N - 1$?

$$W^{(k_1)}(z)W^{(k_2)}(0) = \frac{\delta_{12}}{z^{2k_1}} + \sum_{p=|k_{12}|+2}^{k_1+k_2-2} C_{k_1k_2p} \sum_{m=0}^{k_1+k_2-p-1} \frac{(k_{12}+p)_m}{m!(2p)_m} \frac{\partial^m W^{(p)}(0)}{z^{k_1+k_2-p-m}} + \dots$$

Gives crossing symmetric 4pt function for $k_2 \leq k_1 \leq k_3 \leq k_4$.

$$\frac{\mathcal{F}_{1234}(\chi)}{\chi^{k_1+k_2}} = \sum_{p=max(|k_{12}|,|k_{34}|)+2}^{k_1+k_2-2} C_{k_1k_2p}C_{k_3k_4p} \sum_{m=0}^{k_1+k_2-p-1} \frac{(p-k_{12})_m(p+k_{34})_m}{m!(2p)_m\chi^{k_1+k_2-p-m}} + (s \leftrightarrow t)$$

Should agree with 6d correlator under $\alpha = \alpha' = \frac{1}{\chi'}$.

$$\frac{\mathcal{G}\left(\frac{U}{V}, \frac{1}{V}; \tau, \sigma\right)}{(\sigma U^2)^{\mathcal{E}-\frac{1}{2}(k_1+k_2)}} = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} U^{\frac{s}{2}} V^{k_1+k_4-\frac{s+t}{2}} \sigma^{\frac{1}{2}(k_1+k_2)-\mathcal{E}} \mathcal{M}(s,t;\tau,\sigma)\Gamma_{\{k_i\}}$$

$$\propto \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} (1-\chi')^{\frac{u}{2}-k_2-k_3} \sum_{0 \leq i+j \leq \mathcal{E}} \sigma^i (1-\chi')^{2j} \dots$$

$\chi' = 1$ selects lowest double-trace u pole and no τ.

4d compared to 6d

- Analogue of $W^{(k)}$ with $h = k$ is $J^{(k)}$, $T^{(k)}$ with $(h, j) = \left(\frac{k}{2}, \frac{k}{2}\right), \left(\frac{k+2}{2}, \frac{k-2}{2}\right)$.

4d compared to 6d

- Analogue of $W^{(k)}$ with $h = k$ is $J^{(k)}$, $T^{(k)}$ with $(h, j) = \left(\frac{k}{2}, \frac{k}{2}\right), \left(\frac{k+2}{2}, \frac{k-2}{2}\right)$.
- Analogue of $\mathfrak{sl}(N)$ is $Vect_0(SL(2; \mathbb{C}))$ [Costello, Gaiotto; 1812.09257] .

4d compared to 6d

- Analogue of $W^{(k)}$ with $h = k$ is $J^{(k)}$, $T^{(k)}$ with $(h, j) = \left(\frac{k}{2}, \frac{k}{2}\right), \left(\frac{k+2}{2}, \frac{k-2}{2}\right)$.
- Analogue of $\mathfrak{sl}(N)$ is $Vect_0(SL(2;\mathbb{C}))$ [Costello, Gaiotto; 1812.09257] .
- Each χ block comes with $g_{-j}^{j_{21},j_{43}}\left(\frac{1}{\alpha}\right)$.
- Matches $\mathcal{G}(U, V; \sigma, \tau)$ where we set $\alpha' = \frac{1}{\chi'}$ and leave α.

4d compared to 6d

- Analogue of $W^{(k)}$ with $h = k$ is $J^{(k)}$, $T^{(k)}$ with $(h, j) = \left(\frac{k}{2}, \frac{k}{2}\right), \left(\frac{k+2}{2}, \frac{k-2}{2}\right)$.
- Analogue of $\mathfrak{sl}(N)$ is $Vect_0(SL(2;\mathbb{C}))$ [Costello, Gaiotto; 1812.09257] .
- Each χ block comes with $g^{j_{21},j_{43}}_{-j}\left(\frac{1}{\alpha}\right)$.
- Matches $\mathcal{G}(U, V; \sigma, \tau)$ where we set $\alpha' = \frac{1}{\chi'}$ and leave α.

Learn that $\mathcal{F}_{2222}(\chi;\alpha)\Big|_{1/c^2} = 0$ from simplest OPE.

$$J^{(2)}_{ab}(z)J^{(2)}_{cd}(0) = \frac{1}{z^2} + \frac{\epsilon_{ac}J^{(2)}_{bd}(0) + \epsilon_{ad}J^{(2)}_{cb}(0) + (a \leftrightarrow b)}{cz} + \dots$$

4d compared to 6d

- Analogue of $W^{(k)}$ with $h = k$ is $J^{(k)}$, $T^{(k)}$ with $(h, j) = \left(\frac{k}{2}, \frac{k}{2}\right), \left(\frac{k+2}{2}, \frac{k-2}{2}\right)$.
- Analogue of $\mathfrak{sl}(N)$ is $Vect_0(SL(2;\mathbb{C}))$ [Costello, Gaiotto; 1812.09257] .
- Each χ block comes with $g_{-j}^{j_{21},j_{43}}\left(\frac{1}{\alpha}\right)$.
- Matches $\mathcal{G}(U, V; \sigma, \tau)$ where we set $\alpha' = \frac{1}{\chi'}$ and leave α.

Learn that $\mathcal{F}_{2222}(\chi;\alpha)\Big|_{1/c^2} = 0$ from simplest OPE.

$$J^{(2)}_{ab}(z)J^{(2)}_{cd}(0) = \frac{1}{z^2} + \frac{\epsilon_{ac}J^{(2)}_{bd}(0) + \epsilon_{ad}J^{(2)}_{cb}(0) + (a \leftrightarrow b)}{cz} + \dots$$

$J^{(2)}J^{(2)}$ operators contribute to $J^{(3)} \times J^{(3)} \sim \frac{1}{z^3}$ at $\frac{1}{c^2 z}$.

4d compared to 6d

- Analogue of $W^{(k)}$ with $h = k$ is $J^{(k)}$, $T^{(k)}$ with $(h, j) = \left(\frac{k}{2}, \frac{k}{2}\right), \left(\frac{k+2}{2}, \frac{k-2}{2}\right)$.
- Analogue of $\mathfrak{sl}(N)$ is $Vect_0(SL(2;\mathbb{C}))$ [Costello, Gaiotto; 1812.09257] .
- Each χ block comes with $g^{j_{21},j_{43}}_{-j}\left(\frac{1}{\alpha}\right)$.
- Matches $\mathcal{G}(U, V; \sigma, \tau)$ where we set $\alpha' = \frac{1}{\chi'}$ and leave α.

Learn that $\mathcal{F}_{2222}(\chi;\alpha)\Big|_{1/c^2} = 0$ from simplest OPE.

$$J^{(2)}_{ab}(z)J^{(2)}_{cd}(0) = \frac{1}{z^2} + \frac{\epsilon_{ac}J^{(2)}_{bd}(0) + \epsilon_{ad}J^{(2)}_{cb}(0) + (a \leftrightarrow b)}{cz} + \dots$$

$J^{(2)}J^{(2)}$ operators contribute to $J^{(3)} \times J^{(3)} \sim \frac{1}{z^3}$ at $\frac{1}{c^2 z}$.

$$\mathcal{F}_{3333}(\chi;\alpha)\Big|_{1/c^2} = \frac{2025\chi^2\alpha(1-\alpha)}{c^2(1-\chi)}$$

Plan

1. Half-BPS 4pt functions
   - Superconformal kinematics
   - Holographic correlators in Mellin space
   - How to bootstrap them
2. Checks of chiral symmetry
   - $\mathcal{W}_N$ 4pt functions at tree level
   - Matching to 6d results
   - What changes in 4d
3. Exploring the topological sector
   - OPE coefficients from Mellin space
   - Finite crossing equations
   - Comparisons to matrix model results

New aspects of topological correlators

Techniques based on meromorphy do not work anymore.

$$\langle \mathcal{O}_k(x_1, t_1) \mathcal{O}_k(x_2, t_2) \rangle = \frac{t_{12}^k}{|x_{12}|^k} \mapsto \frac{y_{12}^k}{sgn(x_{12})^k}$$

New aspects of topological correlators

Techniques based on meromorphy do not work anymore.

$$\langle \mathcal{O}_k(x_1, t_1)\mathcal{O}_k(x_2, t_2)\rangle = \frac{t_{12}^k}{|x_{12}|^k} \mapsto \frac{y_{12}^k}{sgn(x_{12})^k}$$

Focus on finite set of OPE coefficients in $\langle \mathcal{O}_2\mathcal{O}_k\mathcal{O}_2\mathcal{O}_k\rangle$.

New aspects of topological correlators

Techniques based on meromorphy do not work anymore.

$$\langle \mathcal{O}_k(x_1, t_1)\mathcal{O}_k(x_2, t_2)\rangle = \frac{t_{12}^k}{|x_{12}|^k} \mapsto \frac{y_{12}^k}{sgn(x_{12})^k}$$

Focus on finite set of OPE coefficients in $\langle \mathcal{O}_2\mathcal{O}_k\mathcal{O}_2\mathcal{O}_k\rangle$.

$$\begin{aligned}[0,0,2,0] \otimes [0,0,k,0] = [0,0,k+2,0] \oplus [0,1,k,0] \oplus [0,2,k-2,0] \\ \oplus [0,0,k,0] \oplus [0,1,k-2,0] \oplus [0,0,k-2,0]\end{aligned}$$

Project onto $Y_{i,j}(\sigma, \tau)$ with $i + j \leq 2$ [Nirschl, Osborn; hep-th/0407060] .

New aspects of topological correlators

Techniques based on meromorphy do not work anymore.

$$\langle \mathcal{O}_k(x_1, t_1)\mathcal{O}_k(x_2, t_2)\rangle = \frac{t_{12}^k}{|x_{12}|^k} \mapsto \frac{y_{12}^k}{sgn(x_{12})^k}$$

Focus on finite set of OPE coefficients in $\langle \mathcal{O}_2\mathcal{O}_k\mathcal{O}_2\mathcal{O}_k\rangle$.

$$\begin{aligned}[0,0,2,0] \otimes [0,0,k,0] = [0,0,k+2,0] \oplus [0,1,k,0] \oplus [0,2,k-2,0] \\ \oplus [0,0,k,0] \oplus [0,1,k-2,0] \oplus [0,0,k-2,0]\end{aligned}$$

Project onto $Y_{i,j}(\sigma,\tau)$ with $i+j \leq 2$ [Nirschl, Osborn; hep-th/0407060] .

$$\lambda^2_{2kB^{[0,0,k,0]}_{\frac{k}{2}}} = \lambda^2_{2kB^{[0,1,k-2,0]}_{\frac{k}{2}}} = \lambda^2_{2kB^{[0,0,k-2,0]}_{\frac{k-2}{2}}} = O(1/c_T)$$

$$\lambda^2_{2kB^{[0,0,k+2,0]}_{\frac{k+2}{2}}} = \frac{2}{(k+1)(k+2)}, \; \lambda^2_{2kB^{[0,1,k,0]}_{\frac{k+2}{2}}} = \frac{2}{k+2}, \; \lambda^2_{2kB^{[0,2,k-2,0]}_{\frac{k+2}{2}}} = \frac{k-1}{k+1}$$

OPE coefficients at tree level

$$[0, a, b, 0] \implies Y_{2a+b-k,a}(\sigma, \tau)$$

$$\ell = 0 \implies V = 1$$

$$\Delta = \frac{k}{2} \implies s = \frac{k}{2} \text{ pole of } \mathcal{M}$$

$$\Delta = \frac{k+2}{2} \implies s = \frac{k+2}{2} \text{ pole of } \Gamma$$

OPE coefficients at tree level

$$[0,a,b,0] \implies Y_{2a+b-k,a}(\sigma,\tau)$$

$$\ell = 0 \implies V = 1$$

$$\Delta = \tfrac{k}{2} \implies s = \tfrac{k}{2} \text{ pole of } \mathcal{M}$$

$$\Delta = \tfrac{k+2}{2} \implies s = \tfrac{k+2}{2} \text{ pole of } \Gamma$$

$$\lambda^2_{2kB^{[0,0,k-2,0]}_{\frac{k-2}{2}}} = 0$$

$$\lambda^2_{2kB^{[0,0,k,0]}_{\frac{k}{2}}} = \frac{32k}{c_T}$$

$$\lambda^2_{2kB^{[0,1,k-2,0]}_{\frac{k}{2}}} = 0$$

OPE coefficients at tree level

$$[0,a,b,0] \implies Y_{2a+b-k,a}(\sigma,\tau)$$
$$\ell = 0 \implies V = 1$$
$$\Delta = \tfrac{k}{2} \implies s = \tfrac{k}{2} \text{ pole of } \mathcal{M}$$
$$\Delta = \tfrac{k+2}{2} \implies s = \tfrac{k+2}{2} \text{ pole of } \Gamma$$

$$\lambda^2_{2kB^{[0,0,k-2,0]}_{\frac{k-2}{2}}} = 0$$
$$\lambda^2_{2kB^{[0,0,k,0]}_{\frac{k}{2}}} = \frac{32k}{c_T}$$
$$\lambda^2_{2kB^{[0,1,k-2,0]}_{\frac{k}{2}}} = 0$$

$$\int_{-i\infty}^{i\infty} \frac{dt}{2\pi i} \frac{\Gamma[a_1 - \frac{t}{2}]\Gamma[a_2 - \frac{t}{2}]\Gamma[b_1 + \frac{t}{2}]\Gamma[b_2 + \frac{t}{2}]}{t \pm 2m - \delta} = \frac{\prod_{i,j=1}^2 \Gamma[a_i + b_j]}{\Gamma[a_1 + a_2 + b_1 + b_2]}$$
$$\times \begin{cases} [a_1 + m + \frac{\delta}{2}]^{-1} {}_3F_2 \left(\begin{matrix} 1, a_1 + b_1, a_1 + b_2 \\ a_1 + a_2 + b_1 + b_2, 1 + a_1 + m + \frac{\delta}{2} \end{matrix} \right) \\ -[b_1 + m + \frac{\delta}{2}]^{-1} {}_3F_2 \left(\begin{matrix} 1, a_1 + b_1, a_2 + b_1 \\ a_1 + a_2 + b_1 + b_2, 1 + b_1 + m + \frac{\delta}{2} \end{matrix} \right) \end{cases}$$

OPE coefficients at tree level

$$[0,a,b,0] \implies Y_{2a+b-k,a}(\sigma,\tau)$$
$$\ell = 0 \implies V = 1$$
$$\Delta = \tfrac{k}{2} \implies s = \tfrac{k}{2} \text{ pole of } \mathcal{M}$$
$$\Delta = \tfrac{k+2}{2} \implies s = \tfrac{k+2}{2} \text{ pole of } \Gamma$$

$$\lambda^2_{2kB^{[0,0,k-2,0]}_{\frac{k-2}{2}}} = 0$$
$$\lambda^2_{2kB^{[0,0,k,0]}_{\frac{k}{2}}} = \frac{32k}{c_T}$$
$$\lambda^2_{2kB^{[0,1,k-2,0]}_{\frac{k}{2}}} = 0$$

$$\int_{-i\infty}^{i\infty} \frac{dt}{2\pi i} \frac{\Gamma[a_1 - \frac{t}{2}]\Gamma[a_2 - \frac{t}{2}]\Gamma[b_1 + \frac{t}{2}]\Gamma[b_2 + \frac{t}{2}]}{t \pm 2m - \delta} = \frac{\prod_{i,j=1}^2 \Gamma[a_i + b_j]}{\Gamma[a_1 + a_2 + b_1 + b_2]}$$
$$\times \begin{cases} [a_1 + m + \frac{\delta}{2}]^{-1} {}_3F_2 \left(\begin{matrix} 1, a_1 + b_1, a_1 + b_2 \\ a_1 + a_2 + b_1 + b_2, 1 + a_1 + m + \frac{\delta}{2} \end{matrix} \right) \\ -[b_1 + m + \frac{\delta}{2}]^{-1} {}_3F_2 \left(\begin{matrix} 1, a_1 + b_1, a_2 + b_1 \\ a_1 + a_2 + b_1 + b_2, 1 + b_1 + m + \frac{\delta}{2} \end{matrix} \right) \end{cases}$$

Sum over m is a 1d crossing kernel [Gopakumar, Sinha; 1809.10975] .

OPE coefficients at tree level

$$I = \sum_{m=0}^{\infty} \frac{(a_1)_m (a_2)_m}{m! (b_1)_m (c+m-1)} {}_3F_2 \left(\begin{matrix} 1, a_3, a_4 \\ b_2, c+m \end{matrix} \right) \propto \sum_{n=0}^{\infty} \binom{a_1 + a_3 - c}{n}$$

$$W(a_2 + b_2 - 1, n+1, a_3, b_2 - a_4, a_2, a_1 + a_2 - b_1 + b_2 - n - 1)$$

Mellin amplitudes are such that $a_1 + a_3 - c \in \mathbb{N}$.

OPE coefficients at tree level

$$I = \sum_{m=0}^{\infty} \frac{(a_1)_m (a_2)_m}{m!(b_1)_m (c+m-1)} {}_3F_2\begin{pmatrix} 1, a_3, a_4 \\ b_2, c+m \end{pmatrix} \propto \sum_{n=0}^{\infty} \begin{pmatrix} a_1 + a_3 - c \\ n \end{pmatrix}$$
$$W(a_2 + b_2 - 1, n+1, a_3, b_2 - a_4, a_2, a_1 + a_2 - b_1 + b_2 - n - 1)$$

Mellin amplitudes are such that $a_1 + a_3 - c \in \mathbb{N}$.

$$W\begin{pmatrix} a, b, c, \\ d, e, f \end{pmatrix} = \int_{-i\infty}^{i\infty} \frac{ds}{2\pi i} \frac{\Gamma[-s]\Gamma[b+s]\Gamma[c+s]\Gamma[d+s]\Gamma[1+a-e-f+s]}{\Gamma[1+a-b-c-d-s]^{-1}\Gamma[1+a-e+s]\Gamma[1+a-f+s]}$$
$$\propto {}_7F_6\begin{pmatrix} a, 1+\frac{1}{2}a, b, c, d, e, f \\ \frac{1}{2}a, 1+a-b, 1+a-c, 1+a-d, 1+a-e, 1+a-f \end{pmatrix}$$

OPE coefficients at tree level

$$I = \sum_{m=0}^{\infty} \frac{(a_1)_m (a_2)_m}{m!(b_1)_m (c+m-1)} {}_3F_2\begin{pmatrix} 1, a_3, a_4 \\ b_2, c+m \end{pmatrix} \propto \sum_{n=0}^{\infty} \begin{pmatrix} a_1 + a_3 - c \\ n \end{pmatrix}$$

$$W(a_2 + b_2 - 1, n+1, a_3, b_2 - a_4, a_2, a_1 + a_2 - b_1 + b_2 - n - 1)$$

Mellin amplitudes are such that $a_1 + a_3 - c \in \mathbb{N}$.

$$W\begin{pmatrix} a, b, c, \\ d, e, f \end{pmatrix} = \int_{-i\infty}^{i\infty} \frac{ds}{2\pi i} \frac{\Gamma[-s]\Gamma[b+s]\Gamma[c+s]\Gamma[d+s]\Gamma[1+a-e-f+s]}{\Gamma[1+a-b-c-d-s]^{-1}\Gamma[1+a-e+s]\Gamma[1+a-f+s]}$$

$$\propto {}_7F_6\begin{pmatrix} a, 1+\frac{1}{2}a, b, c, d, e, f \\ \frac{1}{2}a, 1+a-b, 1+a-c, 1+a-d, 1+a-e, 1+a-f \end{pmatrix}$$

The hypergeometric function happens to have a closed form.

$$\lambda^2_{2kB^{[0,2,k-2,0]}_{\frac{k+2}{2}}} = \frac{16(k-1)}{k^2(k+1)\pi^2 c_T}\left[4(k^3+k^2+2k+4) - k^2(k+2)\left(\pi^2 + 2\psi^{(1)}\left(\frac{k}{2}\right)\right)\right]$$

$$\lambda^2_{2kB^{[0,1,k,0]}_{\frac{k+2}{2}}} = \frac{64}{k^2(k+2)\pi^2 c_T}\left[2(k-1)(k^2-4) - k^2\pi^2 + k^2(k^2+2k-2)\psi^{(1)}\left(\frac{k}{2}\right)\right]$$

$$\lambda^2_{2kB^{[0,0,k+2,0]}_{\frac{k+2}{2}}} = \frac{32}{k(k+1)(k+2)\pi^2 c_T}\left[4(k-1)(k+2) + k^2\pi^2 + 2k^2\psi^{(1)}\left(\frac{k}{2}\right)\right]$$

A finite sum rule

Topological theory still has an OPE [Chester, Lee, Pufu, Yacoby; 1412.0334] .

$$\langle \mathcal{O}_1(\varphi_1, y_1)\mathcal{O}_2(\varphi_2, y_2)\mathcal{O}_3(\varphi_3, y_3)\mathcal{O}_4(\varphi_4, y_4)\rangle = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}} \left(\frac{\alpha}{\alpha - 1}\right)^{j_{43}}$$

$$g_{-j}^{j_{12},j_{43}}\left(\frac{1}{1-\alpha}\right)\left(\frac{y_{14}}{y_{24}}\right)^{j_{12}}\left(\frac{y_{13}}{y_{14}}\right)^{j_{34}}\frac{(-1)^j y_{12}^{j_1+j_2} y_{34}^{j_3+j_4}}{(sgn\varphi_{21})^{\Delta_1+\Delta_2-\Delta}(sgn\varphi_{43})^{\Delta_3+\Delta_4-\Delta}}$$

A finite sum rule

Topological theory still has an OPE [Chester, Lee, Pufu, Yacoby; 1412.0334] .

$$\langle \mathcal{O}_1(\varphi_1, y_1)\mathcal{O}_2(\varphi_2, y_2)\mathcal{O}_3(\varphi_3, y_3)\mathcal{O}_4(\varphi_4, y_4)\rangle = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}} \left(\frac{\alpha}{\alpha - 1}\right)^{j_{43}}$$

$$g_{-j}^{j_{12},j_{43}}\left(\frac{1}{1-\alpha}\right)\left(\frac{y_{14}}{y_{24}}\right)^{j_{12}}\left(\frac{y_{13}}{y_{14}}\right)^{j_{34}} \frac{(-1)^j y_{12}^{j_1+j_2} y_{34}^{j_3+j_4}}{(sgn\varphi_{21})^{\Delta_1+\Delta_2-\Delta}(sgn\varphi_{43})^{\Delta_3+\Delta_4-\Delta}}$$

Apply crossing with $j + j_{34} = 2$ once, 1 twice and 0 thrice.

$$\lambda^2_{2kB^{[0,0,k-2,0]}_{\frac{k-2}{2}}} + \lambda^2_{2kB^{[0,1,k-2,0]}_{\frac{k}{2}}} + \lambda^2_{2kB^{[0,0,k,0]}_{\frac{k}{2}}}$$

$$\lambda^2_{2kB^{[0,2,k-2,0]}_{\frac{k+2}{2}}} + \lambda^2_{2kB^{[0,1,k,0]}_{\frac{k+2}{2}}} - \frac{k(k+3)}{2}\lambda^2_{2kB^{[0,0,k+2,0]}_{\frac{k+2}{2}}} = 0$$

A finite sum rule

Topological theory still has an OPE [Chester, Lee, Pufu, Yacoby; 1412.0334] .

$$\langle \mathcal{O}_1(\varphi_1, y_1)\mathcal{O}_2(\varphi_2, y_2)\mathcal{O}_3(\varphi_3, y_3)\mathcal{O}_4(\varphi_4, y_4)\rangle = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}} \left(\frac{\alpha}{\alpha - 1}\right)^{j_{43}}$$

$$g_{-j}^{j_{12},j_{43}}\left(\frac{1}{1-\alpha}\right)\left(\frac{y_{14}}{y_{24}}\right)^{j_{12}}\left(\frac{y_{13}}{y_{14}}\right)^{j_{34}} \frac{(-1)^j y_{12}^{j_1+j_2} y_{34}^{j_3+j_4}}{(sgn\varphi_{21})^{\Delta_1+\Delta_2-\Delta}(sgn\varphi_{43})^{\Delta_3+\Delta_4-\Delta}}$$

Apply crossing with $j + j_{34} = 2$ once, 1 twice and 0 thrice.

$$\lambda^2_{2kB^{[0,0,k-2,0]}_{\frac{k-2}{2}}} + \lambda^2_{2kB^{[0,1,k-2,0]}_{\frac{k}{2}}} + \lambda^2_{2kB^{[0,0,k,0]}_{\frac{k}{2}}}$$

$$\lambda^2_{2kB^{[0,2,k-2,0]}_{\frac{k+2}{2}}} + \lambda^2_{2kB^{[0,1,k,0]}_{\frac{k+2}{2}}} - \frac{k(k+3)}{2}\lambda^2_{2kB^{[0,0,k+2,0]}_{\frac{k+2}{2}}} = 0$$

Can also constrain $\lambda_{22B_2^{[0040]}}\lambda_{kkB_2^{[0040]}}$, $\lambda_{22B_1^{[0020]}}\lambda_{kkB_1^{[0020]}}$, $\lambda_{22B_2^{[0200]}}\lambda_{kkB_2^{[0200]}}$.

At higher extremality we have $\left\lceil \frac{\mathcal{E}}{2} \right\rceil$ crossing equations.

Comparing to a matrix model

TQFT for fundamental and adjoint hyper [Dedushenko, Pufu, Yacoby; 1610.00740] .

$$S_Q = -\int_{-\pi}^{\pi} d\varphi \tilde{Q}_\alpha [\dot{Q} + \sigma Q]^\alpha, \quad S_X = -\int_{-\pi}^{\pi} d\varphi \tilde{X}^\alpha{}_\beta \left[\dot{X} + [\sigma, X] \right]^\beta{}_\alpha$$

Comparing to a matrix model

TQFT for fundamental and adjoint hyper [Dedushenko, Pufu, Yacoby; 1610.00740] .

$$S_Q = -\int_{-\pi}^{\pi} d\varphi \tilde{Q}_\alpha [\dot{Q} + \sigma Q]^\alpha, \quad S_X = -\int_{-\pi}^{\pi} d\varphi \tilde{X}^\alpha{}_\beta \left[\dot{X} + [\sigma, X]\right]^\beta{}_\alpha$$

Combine with dualities for ABJM at level 1 [Bashkirov, Kapustin; 1103.3548] .

Comparing to a matrix model

TQFT for fundamental and adjoint hyper [Dedushenko, Pufu, Yacoby; 1610.00740] .

$$S_Q = -\int_{-\pi}^{\pi} d\varphi \tilde{Q}_\alpha [\dot{Q} + \sigma Q]^\alpha, \quad S_X = -\int_{-\pi}^{\pi} d\varphi \tilde{X}^\alpha{}_\beta \left[\dot{X} + [\sigma, X]\right]^\beta{}_\alpha$$

Combine with dualities for ABJM at level 1 [Bashkirov, Kapustin; 1103.3548] .

Correlators for multi-trace $\mathcal{X} = (X, \tilde{X})$ operators.

$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = \frac{1}{N! Z_N} \int d^N\sigma \prod_{\alpha<\beta} 4sinh^2(\pi\sigma_{\alpha\beta}) Z_\sigma \langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle_\sigma$$

$$\left\langle \mathcal{X}^\alpha{}_\beta(\varphi_1, y_1) \mathcal{X}^\delta{}_\gamma(\varphi_2, y_2) \right\rangle_\sigma = y_{12} \delta^\alpha_\gamma \delta^\delta_\beta \frac{sgn\varphi_{12} + tanh(\pi\sigma_{\alpha\beta})}{2e^{-\sigma_{\alpha\beta}\varphi_{12}}}$$

Comparing to a matrix model

TQFT for fundamental and adjoint hyper [Dedushenko, Pufu, Yacoby; 1610.00740] .

$$S_Q = -\int_{-\pi}^{\pi} d\varphi \tilde{Q}_\alpha [\dot{Q} + \sigma Q]^\alpha, \quad S_X = -\int_{-\pi}^{\pi} d\varphi \tilde{X}^\alpha{}_\beta \left[\dot{X} + [\sigma, X]\right]^\beta{}_\alpha$$

Combine with dualities for ABJM at level 1 [Bashkirov, Kapustin; 1103.3548] .

Correlators for multi-trace $\mathcal{X} = (X, \tilde{X})$ operators.

$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = \frac{1}{N! Z_N} \int d^N\sigma \prod_{\alpha < \beta} 4 sinh^2(\pi \sigma_{\alpha\beta}) Z_\sigma \langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle_\sigma$$

$$\left\langle \mathcal{X}^\alpha{}_\beta(\varphi_1, y_1) \mathcal{X}^\delta{}_\gamma(\varphi_2, y_2) \right\rangle_\sigma = y_{12} \delta^\alpha_\gamma \delta^\delta_\beta \frac{sgn \varphi_{12} + tanh(\pi \sigma_{\alpha\beta})}{2e^{-\sigma_{\alpha\beta}\varphi_{12}}}$$

Leading $(O(1/c_T))$ single-trace couplings match [Mezei, Pufu, Wang; 1703.08749] .

Comparing to a matrix model

TQFT for fundamental and adjoint hyper [Dedushenko, Pufu, Yacoby; 1610.00740] .

$$S_Q = -\int_{-\pi}^{\pi} d\varphi \tilde{Q}_\alpha [\dot{Q} + \sigma Q]^\alpha, \quad S_X = -\int_{-\pi}^{\pi} d\varphi \tilde{X}^\alpha{}_\beta \left[\dot{X} + [\sigma, X]\right]^\beta{}_\alpha$$

Combine with dualities for ABJM at level 1 [Bashkirov, Kapustin; 1103.3548] .

Correlators for multi-trace $\mathcal{X} = (X, \tilde{X})$ operators.

$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = \frac{1}{N! Z_N} \int d^N\sigma \prod_{\alpha<\beta} 4 sinh^2(\pi\sigma_{\alpha\beta}) Z_\sigma \langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle_\sigma$$

$$\left\langle \mathcal{X}^\alpha{}_\beta(\varphi_1, y_1) \mathcal{X}^\delta{}_\gamma(\varphi_2, y_2) \right\rangle_\sigma = y_{12} \delta^\alpha_\gamma \delta^\delta_\beta \frac{sgn\varphi_{12} + tanh(\pi\sigma_{\alpha\beta})}{2e^{-\sigma_{\alpha\beta}\varphi_{12}}}$$

Leading ($O(1/c_T)$) single-trace couplings match [Mezei, Pufu, Wang; 1703.08749] .

Leading ($O(1)$) double-trace OPE coefficients do too.

$$\lambda^2_{\mathcal{E} k [\mathcal{O}_\mathcal{E} \mathcal{O}_k]^{[0,j,k+\mathcal{E}-2j,0]}} = \frac{k! \mathcal{E}!}{j!} \frac{k + \mathcal{E} - 2j + 1}{(k + \mathcal{E} - j + 1)!}$$

Conclusions

- Mellin amplitudes for $AdS_7 \times S^4$, $AdS_5 \times S^5$ and $AdS_4 \times S^7$ enable important checks.
- Many previously disparate results are special cases of the crossing kernel.
- The chiral algebra structure makes predictions about loops from AdS unitarity method [Aharony, Alday, Bissi, Perlmutter; 1612.03891] .
- Closed form OPE coefficients in ABJM theory present a challenge for matrix model techniques [Mariño, Putrov; 1110.4066] .
- Protected operators in 3d $\mathcal{N} \geq 4$ and 4d $\mathcal{N} \geq 2$ SCFTs allow us to study similar conjectures [Binder, Chester, Pufu; 1906.07195] .
- Should also explore backgrounds with defects or finite temperature.