Hidden structures of holographic correlators

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2021-03-25

[2101.04114] with P. Ferrero, X. Zhou
[2103.xxxxx] with L. F. Alday, P. Ferrero, X. Zhou

Types of hidden symmetries

	Chiral	Hidden conformal	Parisi-Sourlas
3d $\mathcal{N} = 8$ ABJM			
4d $\mathcal{N} = 4$ SYM			
6d $\mathcal{N} = (2, 0)$			
3d $\mathcal{N} = 3$ flavoured ABJM			
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How tractable is half-maximal SUSY?

Simplest two selection rules for KK-modes in $\mathcal{N} = 4$ SYM:

$$\mathcal{B}_{[0,2,0]} \times \mathcal{B}_{[0,2,0]} = \mathcal{B}_{[0,2,0]}$$
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Decomposition into $\mathcal{N} = 2$ multiplets: [Dolan, Osborn; hep-th/0209056]

$$\mathcal{B}_{[0,2,0]} = 3\hat{\mathcal{B}}_1 + \mathcal{E}_{\pm 2} + \hat{\mathcal{C}}_0 + 2\mathcal{D}_{\pm \frac{1}{2}}$$

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- We will break supersymmetry with space-filling branes.
- Theories are "open string analogues" of ones with maximal SUSY.
- External ops will be currents for G_F — not possible with 16 Qs.

Example setup in four dimensions

Stack of $N \gg 1$ D3-branes has $AdS_5 \times S^5$ near-horizon geometry.

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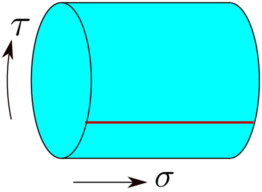
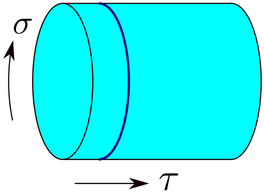


D3-D3 open strings



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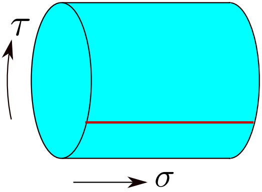

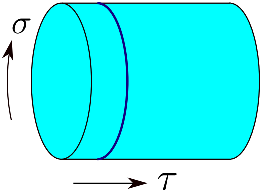

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D3-D7 / D7-D3 open strings

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	0	1	2	3	4	5	6	7	8	9
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D7	x	x	x	x	x	x	x	x		

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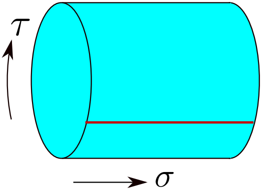

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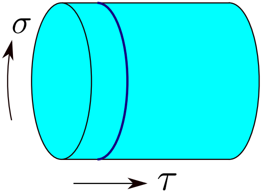

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KK-modes of 8d Yang-Mills on S^3 involve only short multiplets!

Similar idea in other dimensions

$$AdS_7 : SO(5)_R \to SU(2)_L \times SU(2)_R$$
$$AdS_6 : SO(5)_R \to SU(2)_L \times SU(2)_R$$
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Set $\epsilon = \frac{d-2}{2}$ and consider half-BPS external ops with $\Delta = \epsilon k$.

$$\mathcal{O}^I_k(x, v, \bar{v}) = \mathcal{O}^I_{\alpha_1 \dots \alpha_k ; \bar{\alpha}_1 \dots \bar{\alpha}_{k-2}} v^{\alpha_1} \dots v^{\alpha_k} \bar{v}^{\bar{\alpha}_1} \dots \bar{v}^{\bar{\alpha}_{k-2}}$$

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4pt function is degree k in $\alpha = \frac{v_{13} v_{24}}{v_{12} v_{34}}$ and k - 2 in $\beta = \frac{\bar{v}_{13} \bar{v}_{24}}{\bar{v}_{12} \bar{v}_{34}}$.

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More generally $\mathcal{E}$ which is either $min(k_i)$ or $\frac{1}{2} \sum k_i - min(k_i)$.

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We can write ansatz for $\mathcal{G}^{I_1 I_2 I_3 I_4}(z, \bar{z}; \alpha, \beta)$ in position or $\mathcal{M}^{I_1 I_2 I_3 I_4}(s, t; \alpha, \beta)$ in Mellin space but how do we fix coefficients?

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Four (two) identities for 16 Qs (8 Qs) [Dolan, Gallot, Sokatchev; hep-th/0405180] .

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Exploit $z \leftrightarrow \bar{z}$ and write $z^q \pm \bar{z}^q$ in terms of U, V [Zhou; 1712.02800] .

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$$U\partial_U \to \left[\frac{s}{2} - \epsilon \frac{k_1 + k_2 - 2\mathcal{E}}{2} \right] \times, \quad V\partial_V \to \left[\frac{t}{2} + \epsilon \frac{k_1 - k_4 - 2\mathcal{E}}{2} \right] \times$$

$$U^m V^n \circ \mathcal{M}(s,t) = \left(\frac{\Delta_1 + \Delta_2 - s}{2} \right)_m \left(\frac{\Delta_3 + \Delta_4 - s}{2} \right)_m \left(\frac{\Delta_1 + \Delta_4 - t}{2} \right)_n \left(\frac{\Delta_2 + \Delta_3 - t}{2} \right)_n \left(\frac{\Delta_1 + \Delta_3 - u}{2} \right)_{-m-n} \left(\frac{\Delta_2 + \Delta_4 - u}{2} \right)_{-m-n} \mathcal{M}(s-2m, t-2n)$$

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Same as setting $v_i = [1, \bar{z}_i]^T$ [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees; 1312.5344] !

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Second factor gives $\bar{L}_{0,\pm 1} - R_{0,\pm 1} = \{\mathbb{Q}, \cdot\}$ commuting with $L_{0,\pm 1}$.

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4d $\mathcal{N} = 4$ SYM	$-3(N^2 - 1)$	Analogue for $Vect_0(SL(2;\mathbb{C}))$? [Costello, Gaiotto; 1812.09257]

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4d $\mathcal{N} = 4$ SYM	$-3(N^2 - 1)$	Analogue for $Vect_0(SL(2;\mathbb{C}))$? [Costello, Gaiotto; 1812.09257]
Lowest AD	$-22/5$	Lee-Yang model

I. Chiral algebra / SCFT correspondence

Comes from nilpotent $\mathbb{Q}$ in $\mathfrak{sl}(2) \times \mathfrak{sl}(2|2)$ preserving a plane.
Second factor gives $\bar{L}_{0,\pm 1} - R_{0,\pm 1} = \{\mathbb{Q}, \cdot\}$ commuting with $L_{0,\pm 1}$.

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Whether conjecture is precise or not, use chiral OPE.

$$\mathcal{O}_{h_1}(z)\mathcal{O}_{h_2}(0) = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}} \sum_{m=0}^{\infty} \frac{(h_{12}+h)_m}{m!(2h)_m} \frac{\partial^m \mathcal{O}_h(0)}{z^{h_1+h_2-h-m}}$$

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Structure constants agree with those in full 4d or 6d theory.

The holographic case

Need to solve for missing data.

$$\mathcal{M}_{16Qs}(s, t; \alpha, \bar{\alpha}) = \sum_p C_{12p} C_{34p} \mathcal{S}_p(s, t; \alpha, \bar{\alpha}) + \textit{crossed} + \textit{contact}$$

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For 6d $\mathcal{N}=(2,0)$, use $C_{123} = \sqrt{\frac{35}{2c_T}} \frac{4^\nu \Gamma[\nu]}{\pi^{\frac{3}{2}}} \prod_i \frac{\Gamma\left[\nu_i + \frac{1}{2}\right]}{\sqrt{\Gamma[2k_i - 1]}}$ where $\nu_i = \frac{k_1+k_2+k_3}{2} - k_i$ [Bastianelli, Zucchini; hep-th/9907047] .

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Simplest chiral algebra correlator in 4d $\mathcal{N} = 4$ SYM.

$$F_{2222}(z;\alpha) = 1 + (z\alpha)^2 + z^2\left(\frac{\alpha - 1}{1 - z}\right)^2 + \frac{12}{c}\left[\frac{z}{1 - z} - 2z\alpha + \frac{(z\alpha)^2}{1 - z}\right]$$

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Learn that $F_{2222}(z;\alpha)\big|_{1/c^2} = 0$ from exact OPE.

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$$F_{3333}(z;\alpha)\Big|_{1/c^2} = \frac{2025z^2\alpha(1-\alpha)}{c^2(1-z)}$$

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$$\widetilde{\mathcal{M}}(s,t;\sigma,\tau) = \sum_{i+j+k=\mathcal{E}-2}\frac{a_{ijk}\sigma^i\tau^j}{(s-s_M+2k)(t-t_M+2j)(u-u_M+2i)}$$

$$s_M = min(k_1+k_2, k_3+k_4) - 2$$

$$t_M = min(k_1+k_4, k_2+k_3) - 2$$

$$u_M = min(k_1+k_3, k_2+k_4) + 2$$

$$a_{ijk} = \frac{240\sqrt{k_1k_2k_3k_4}}{c_T\, i!j!k!\left(1+\frac{|k_1+k_3-k_2-k_4|}{2}\right)_i\left(1+\frac{|k_1+k_4-k_2-k_3|}{2}\right)_j\left(1+\frac{|k_1+k_2-k_3-k_4|}{2}\right)_k}$$

II. A more mysterious hidden structure

Conformally flat $AdS_5 \times S^5$ vs actually flat $\mathbb{R}^{10}$.

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Reflects conformal flatness of $AdS_5 \times S^3$ locus for the brane.

Back to the full Mellin amplitude

Residues have degree 2 for gravitons and 1 for gluons.

$$\mathcal{S}_p^{16Qs}(s,t;\sigma,\tau) = \sum_{m=0}^{\infty} \sum_{0 \leq i+j \leq \mathcal{E}} \sigma^i \tau^j \frac{K_p^{ij}(t,u) H_{p,m}^{ij}}{s - \epsilon p - 2m}$$

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Zoom in on polynomial.

$$K_p^i = -2i(2i+\kappa_t)u^+ + 2i(\kappa_u-2)t^+ - \tfrac{1}{4}(t^- - 2i\epsilon)(2p-\kappa_t-\kappa_u)(2p+\kappa_t+\kappa_u-4)$$

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Can pull outside with $(\alpha-1)\partial_\alpha \leftrightarrow -i\times$, etc.

$$\widehat{K}_p = -8(\mathcal{E}-\theta_{12})D_{23} - 2p(p-2)[D_{14} - \epsilon(\mathcal{E}-\theta_{12})] - 4(\mathcal{E}-\theta_{12})^2(D_{13}+D_{24})$$

$$\theta_{ij} = v_{ij}\frac{\partial}{\partial v_{ij}}, \quad D_{ij} = x_{ij}^2 \frac{\partial}{\partial x_{ij}^2}$$

An old conjecture

Actions for different d are perturbatively equivalent [Parisi, Sourlas; 1979] .

$$S = \int d^d x d\theta d\bar{\theta} \left[-\frac{1}{2} \Phi \partial^2 \Phi + V(\Phi) \right] \leftrightarrow S = \int d^{d-2} x \left[-\frac{1}{2} \phi \partial^2 \phi + V(\phi) \right]$$

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LHS invariant under $\mathfrak{osp}(d+1,1|2)$ superalgebra.

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Superblock can be expressed in two ways [Kaviraj, Rychkov, Trevisani; 1912.01617] .

$$G^{(d-2)}_{\Delta,\ell} = G^{(d)}_{\Delta,\ell} + c_{2,0} G^{(d)}_{\Delta+2,\ell} + c_{1,-1} G^{(d)}_{\Delta+1,\ell-1} + c_{0,-2} G^{(d)}_{\Delta,\ell-2} + c_{2,-2} G^{(d)}_{\Delta+2,\ell-2}$$

An old conjecture

Actions for different d are perturbatively equivalent [Parisi, Sourlas; 1979] .

$$S = \int d^d x d\theta d\bar{\theta} \left[-\frac{1}{2}\Phi\partial^2\Phi + V(\Phi) \right] \leftrightarrow S = \int d^{d-2}x \left[-\frac{1}{2}\phi\partial^2\phi + V(\phi) \right]$$

LHS invariant under $\mathfrak{osp}(d+1,1|2)$ superalgebra.

$$\delta_{\mu\nu} \to \delta_{\mu\nu} \otimes \Omega_{+-}$$
$$P_\mu \to (P_\mu, Q_+, Q_-), \quad K_\mu \to (K_\mu, S_+, S_-)$$
$$\textit{Similar for } M_{\mu\nu}$$

Superblock can be expressed in two ways [Kaviraj, Rychkov, Trevisani; 1912.01617] .

$$G^{(d-2)}_{\Delta,\ell} = G^{(d)}_{\Delta,\ell} + c_{2,0}G^{(d)}_{\Delta+2,\ell} + c_{1,-1}G^{(d)}_{\Delta+1,\ell-1} + c_{0,-2}G^{(d)}_{\Delta,\ell-2} + c_{2,-2}G^{(d)}_{\Delta+2,\ell-2}$$

Applies to Witten diagrams as well [Zhou; 2005.03031] .

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III. Parisi-Sourlas SUSY in holography

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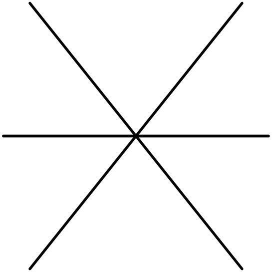
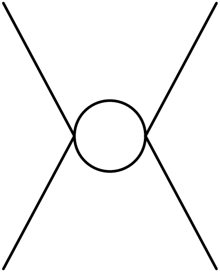
$$\mathcal{S}_p(x_i, t_i) = C(k_i, p)\widehat{K}_p \circ \left[W^{(d-\#Qs/4)}_{\epsilon p,0}(x_i) Y^{(n-\#Qs/4)}_p(t_i)\right]$$

Future questions





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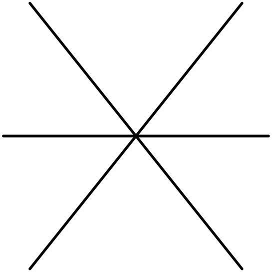

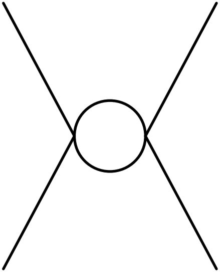

- Higher derivative corrections involve contact terms of growing degree. These give information about "Veneziano amplitude" of AdS [Abl, Heslop, Lipstein; 2012.12091] .
- Possible to consider both gravitons $O(1/c_T)$ and gluons $O(1/c_J)$ to study backreaction on the brane.
- More features of flat space gauge theory amplitudes can now be checked in AdS. These include color-kinematic duality and perhaps the double copy.