Hidden structures of holographic correlators

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2021-08-12

[2101.04114] with P. Ferrero, X. Zhou
[2103.15830] with L. F. Alday, P. Ferrero, X. Zhou

Types of hidden symmetries

	Chiral	Hidden conformal	Parisi-Sourlas
3d $\mathcal{N} = 8$ ABJM			
4d $\mathcal{N} = 4$ SYM			
6d $\mathcal{N} = (2, 0)$			
3d $\mathcal{N} = 3$ flavoured ABJM			
4d $\mathcal{N} = 2$ Argyres-Douglas			
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Kinematics

Consider a theory with $SL(2)$ conformal, $SU(2)$ global symmetry:

$$\mathcal{O}(z, v) = \mathcal{O}^{\alpha_1 \dots \alpha_{2j}}(z) v_{\alpha_1} \dots v_{\alpha_{2j}}.$$

4pt functions depend on cross ratios:

$$\chi = \frac{z_{12} z_{34}}{z_{13} z_{24}}, \quad \alpha = \frac{v_{13} v_{24}}{v_{12} v_{34}}.$$

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$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle = \left(\frac{z_{24}}{z_{14}} \right)^{h_{12}} \left(\frac{z_{14}}{z_{13}} \right)^{h_{34}} \frac{1}{z_{12}^{h_1 + h_2} z_{34}^{h_3 + h_4}} \mathcal{G}(\chi).$$

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Another uses extremality $\mathcal{E} = 2\text{min}(j_i)$ or $\sum_i j_i - 2\text{max}(j_i)$.

$$v_{\overline{34}}^{j_{\bar{3}} + j_{\bar{4}} - j_{\bar{1}} - j_{\bar{2}}} v_{\overline{24}}^{j_{\bar{2}} + j_{\bar{4}} - j_{\bar{1}} - j_{\bar{3}}} v_{\overline{23}}^{j_{\bar{1}} + j_{\bar{2}} + j_{\bar{3}} - j_{\bar{4}} - \mathcal{E}} v_{\overline{14}}^{2 j_{\bar{1}} - \mathcal{E}} (v_{\overline{12}} v_{\overline{34}})^{\mathcal{E}}.$$

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$$\left[\frac{v_{34}}{x_{34}^{2\epsilon}} \right]^{j_3+j_4-j_1-j_2} \left[\frac{v_{24}}{x_{24}^{2\epsilon}} \right]^{j_2+j_4-j_1-j_3} \left[\frac{v_{23}}{x_{23}^{2\epsilon}} \right]^{j_1+j_2+j_3-j_4-\mathcal{E}} \left[\frac{v_{14}}{x_{14}^{2\epsilon}} \right]^{2j_1-\mathcal{E}} \left[\frac{v_{12} v_{34}}{x_{12}^{2\epsilon} x_{34}^{2\epsilon}} \right]^{\mathcal{E}}$$

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For $d > 2$, use $U = \chi\chi'$ and $V = (1-\chi)(1-\chi')$.

Maximal and half-maximal SUSY

With 16 Qs, R symmetry is $SO(5)$, $SO(6)$ or $SO(8)$.

$$\mathcal{O}(x,t) = \mathcal{O}^{I_1 \dots I_k}(x) t_{I_1} \dots t_{I_k}, \quad \Delta = \epsilon k, \quad \epsilon = \frac{d-2}{2}$$

$$\sigma = \frac{t_{13} t_{24}}{t_{12} t_{34}} = \alpha\alpha', \quad \tau = \frac{t_{14} t_{23}}{t_{12} t_{34}} = (1-\alpha)(1-\alpha')$$

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One can use a brane to break this to 8 Qs and $SU(2)_L \times SU(2)_R$.

$$\mathcal{O}(x, v, \bar{v}) = \mathcal{O}^{\alpha_1 \dots \alpha_k}_{\beta_1 \dots \beta_{k-2}}(x) v_{\alpha_1} \dots v_{\alpha_k} \bar{v}^{\beta_1} \dots \bar{v}^{\beta_{k-2}}, \quad \Delta = \epsilon k$$

$$\alpha = \frac{v_{12} v_{34}}{v_{13} v_{34}}, \quad \beta = \frac{\bar{v}_{12} \bar{v}_{34}}{\bar{v}_{13} \bar{v}_{34}}, \quad \sigma = \alpha \beta, \quad \tau = (1-\alpha)(1-\beta)$$

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Four (two) identities for 16 Qs (8 Qs) [Dolan, Gallot, Sokatchev; hep-th/0405180] .

$$(\chi \partial_\chi - \epsilon \alpha \partial_\alpha) \mathcal{G} \big|_{\alpha = \chi^{-1}} = 0$$

Holography in Mellin space

Use Mandelstam variables instead of cross ratios [Mack; 0907.2407] .

$$\mathcal{G}(U,V) = \int_{-i\infty}^{i\infty} \frac{dsdt}{(4\pi i)^2} \frac{V^{\frac{\epsilon}{2}(k_1-k_4-\mathcal{E})+\frac{t}{2}}}{U^{\frac{\epsilon}{2}(k_1+k_2-\mathcal{E})-\frac{s}{2}}} \mathcal{M}(s,t) \Gamma\left[\frac{\Delta_1+\Delta_2-s}{2}\right]\Gamma\left[\frac{\Delta_3+\Delta_4-s}{2}\right]$$
$$\Gamma\left[\frac{\Delta_1+\Delta_4-t}{2}\right]\Gamma\left[\frac{\Delta_2+\Delta_3-t}{2}\right]\Gamma\left[\frac{\Delta_1+\Delta_3-u}{2}\right]\Gamma\left[\frac{\Delta_2+\Delta_4-u}{2}\right]$$

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Conformal blocks and Witten diagrams (for $\tau = \Delta - \ell$) both become

$$\mathcal{M}_{\tau,\ell}(s,t) = \sum_{m=0}^{\infty} \frac{Q^{\tau}_{\ell,m}(t)}{m!\Gamma\left[\frac{\Delta_1+\Delta_2-\tau}{2} - m\right] \Gamma\left[\frac{\Delta_3+\Delta_4-\tau}{2} - m\right] (s-\tau-2m)}.$$

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Add contact terms to single trace blocks in all channels.

$$\mathcal{M}(s,t) = \sum_{\mathcal{O}} C_{12\mathcal{O}}C_{34\mathcal{O}}\mathcal{M}_{\mathcal{O}}(s,t) + C_{14\mathcal{O}}C_{23\mathcal{O}}\mathcal{M}_{\mathcal{O}}(t,s) + C_{13\mathcal{O}}C_{24\mathcal{O}}\mathcal{M}_{\mathcal{O}}(u,t)$$
$$+ P_{\ell_{max}-1}(s,t)$$

Operator content

Instead of $\mathcal{M}_{\mathcal{O}}(s, t)$, use $\mathcal{S}_{\mathcal{O}}(s, t; \alpha)$ which is a linear combination of $\mathcal{Y}_j(\alpha)\mathcal{M}_{\tau,\ell}(s, t)$ for all components of the superconformal block.

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Instead of $\mathcal{M}_{\mathcal{O}}(s,t)$, use $\mathcal{S}_{\mathcal{O}}(s,t;\alpha)$ which is a linear combination of $\mathcal{Y}_j(\alpha)\mathcal{M}_{\tau,\ell}(s,t)$ for all components of the superconformal block.

	$OSp(8\|4)$	$PSU(2,2\|2)$	$OSp(8^*\|4)$
$\ell=0$	$B[0]^{[0,0,p,0]}_{p/2}$	$B\bar{B}[0,0]^{[0,p,0]}_{p}$	$D[0,0,0]^{[p,0]}_{2p}$
$\ell=2$	$Q^4B[0]^{[0,0,p,0]}_{p/2}$	$Q^2\bar{Q}^2B\bar{B}[0,0]^{[0,p,0]}_{p}$	$Q^4D[0,0,0]^{[p,0]}_{2p}$
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With 16 Qs, selection rule is

$$\mathcal{O}_{k_1} \times \mathcal{O}_{k_2} \supset \mathcal{O}_p, \quad p \in \{|k_{12}|+2, |k_{12}|+4, \dots, k_1+k_2-2\}.$$

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Generic non-maximal theories can be messier [Dolan, Osborn; heo-th/0209056] .

$$B\bar{B}[0,0]^{[0,4,0]}_4=B\bar{B}[0,0]^{2,2}_4\oplus A_2\bar{A}_2[0,0]^{1,1}_4\oplus L\bar{L}[0,0]^{0,0}_4\oplus\dots$$

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If SUSY is broken with a brane, look for $\ell=1\Rightarrow$ half-BPS again!

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With 16 Qs and 8 Qs, selection rule is

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Probe branes wrap $AdS_{d+1} \times S^3$ to produce $SU(2)_L \times SU(2)_R$.

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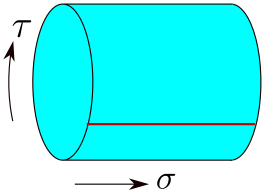
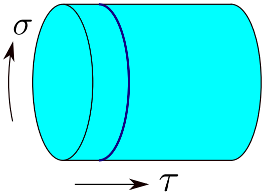


D3-D3 open strings



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Super-gravitons and super-gluons

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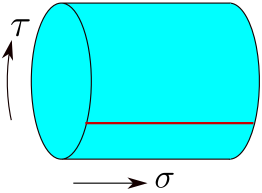
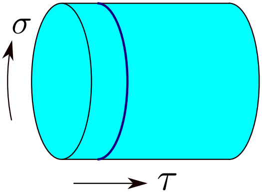


D3-D3 open strings

D3-D7 / D7-D3 open strings



Closed strings

D7-D7 open strings

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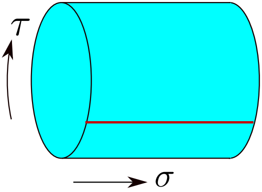

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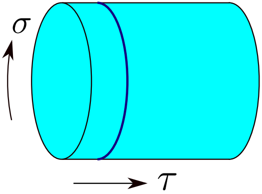

Closed strings

D7-D7 open strings

$$\mathcal{M}_{16Qs}(s,t;\alpha,\alpha') = \sum_{p=|k_{12}|+2}^{k_1+k_2-2} C_{12p}C_{34p}\mathcal{S}_p(s,t;\alpha,\alpha') + \textit{crossed} + \textit{contact}$$

$$\mathcal{M}_{8Qs}^{I_1I_2I_3I_4}(s,t;\alpha,\beta) = f^{I_1I_2J}f^{JI_3I_4}\sum_{p=|k_{12}|+2}^{k_1+k_2-2} C_{12p}C_{34p}\mathcal{S}_p(s,t;\alpha)\mathcal{Y}_{p-2}(\beta) + \textit{same}$$

Scatter $\mathcal{O}_k(x,t)$ at $O(1/c_T)$ and $\mathcal{O}_k^I(x,v,\bar{v})$ at $O(1/c_J)$.

$C_{k_1,k_2,k_3} = ?$

Determined in terms of $\Xi = \frac{1}{2}\sum_i k_i$ and $\alpha_i = \Xi - k_i$.

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$$C^{16Qs}_{k_1,k_2,k_3} = \begin{cases} \sqrt{\frac{70}{\pi^3 c_T}} 2^{2\Xi-1}\Gamma[\Xi]\prod_i \frac{\Gamma\left[\alpha_i + \frac{1}{2}\right]}{\sqrt{\Gamma[2k_i - 1]}} & AdS_7 \\ \sqrt{\frac{30 k_1 k_2 k_3}{c_T}} & AdS_5 \\ \sqrt{\frac{\pi}{3c_T}} \frac{2^{3-\Xi}}{\Gamma\left[\frac{\Xi+2}{2}\right]} \prod_i \frac{\sqrt{\Gamma[k_i+2]}}{\Gamma\left[\frac{\alpha_i+1}{2}\right]} & AdS_4 \end{cases}$$

from [Lee, Minwalla, Rangamani, Seiberg; hep-th/9806074] [Bastianelli, Zucchini; hep-th/9907047] .

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from [Alday, CB, Ferrero, Zhou; 2103.15830] .

1. Superconformal Ward identity

Apply $(\chi\partial_\chi - \epsilon\alpha\partial_\alpha)\,\mathcal{G}\big|_{\alpha=\chi^{-1}} = 0$ in Mellin space [Zhou; 1712.02800] .

$$U\partial_U \to \left[\frac{s}{2} - \epsilon\frac{k_1 + k_2 - 2\mathcal{E}}{2}\right]\times, \quad V\partial_V \to \left[\frac{t}{2} + \epsilon\frac{k_1 - k_4 - 2\mathcal{E}}{2}\right]\times$$

$$U^m V^n \circ \mathcal{M}(s,t) = \left(\frac{\Delta_1 + \Delta_2 - s}{2}\right)_m \left(\frac{\Delta_3 + \Delta_4 - s}{2}\right)_m \left(\frac{\Delta_1 + \Delta_4 - t}{2}\right)_n$$
$$\left(\frac{\Delta_2 + \Delta_3 - t}{2}\right)_n \left(\frac{\Delta_1 + \Delta_3 - u}{2}\right)_{-m-n} \left(\frac{\Delta_2 + \Delta_4 - u}{2}\right)_{-m-n} \mathcal{M}(s - 2m, t - 2n)$$

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Residues of $\mathcal{S}_p$ can have m dependence simplified.

$$\frac{\sigma^i\tau^j H^{i,j}_{p,m}}{s-\epsilon p-2m}\left[t^2 + q_1^p(m)t + q_2^p(m)\right] \quad or \quad \frac{(1-\alpha)^i H^i_{p,m}}{s-\epsilon p-2m}\left[t + q_1^p(m)\right]$$

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Leads to amplitudes with no additional contact terms [Alday, Zhou; 2006.06653] .

2. Flat space limit

Amplitudes take a universal form for $s, t \to \infty$ with $s + t + u = 0$.

$$\frac{\mathcal{M}_{16Qs}(s, t, \alpha, \alpha')}{P_{\mathcal{E}-2}(\sigma, \tau)} = \frac{(s + t - \alpha s)^2 (s + t - \alpha' s)^2}{stu}$$

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With color and kinematic factors

$$\mathsf{c}_s = f^{I_1 I_2 J} f^{J I_3 I_4}, \quad \mathsf{c}_t = f^{I_1 I_4 J} f^{J I_2 I_3}, \quad \mathsf{c}_u = f^{I_1 I_3 J} f^{J I_4 I_2}$$

$$N_s = u(1 - \alpha) - t\alpha, \quad N_t = (\alpha - 1)(u + s\alpha), \quad N_u = \alpha(t + s(1 - \alpha))$$

gluon analogue agrees with [Adamo, Casali, Mason, Nekovar; 1810.05115] :

$$\frac{\mathcal{M}_{8Qs}(s, t, \alpha, \beta)}{P_{\mathcal{E}-2}(\sigma, \tau)} = \frac{(t\mathsf{c}_s - s\mathsf{c}_t)(s + t - \alpha s)^2}{stu} = \left[\frac{\mathsf{c}_s N_s}{s} + \frac{\mathsf{c}_t N_t}{t} + \frac{\mathsf{c}_u N_u}{u} \right].$$

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Polynomial given by the overlap of wavefunctions on a transverse S^d.

$$P_{\mathcal{E}-2}(\sigma,\tau) \propto \int_{S^d} dT (t_1 \cdot T)^{k_1-2}(t_2 \cdot T)^{k_2-2}(t_3 \cdot T)^{k_3-2}(t_4 \cdot T)^{k_4-2}$$

3. Chiral algebra

Superconformal Ward identities in four dimensions are

$$\partial_{\chi'} \mathcal{G}^{\mathcal{N}=2}(\chi, \chi'; \chi'^{-1}) = 0, \quad \partial_{\chi'} \mathcal{G}^{\mathcal{N}=4}(\chi, \chi'; \alpha, \chi'^{-1}) = 0.$$

Solutions have the form $\mathcal{G} = \mathcal{K} + R\mathcal{H}$ with

$$R^{\mathcal{N}=2} = (1 - \alpha\chi)(1 - \alpha\chi'), \quad R^{\mathcal{N}=4} = (1 - \alpha\chi)(1 - \alpha\chi')(1 - \alpha'\chi)(1 - \alpha'\chi').$$

Cohomological explanation: [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees; 1312.5344] .

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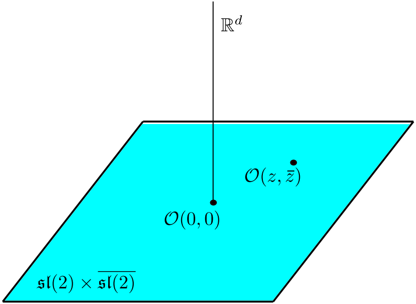

Plane preserved by $\mathfrak{sl}(2) \times \mathfrak{sl}(2|2)$ admits nilpotent $\mathbb{Q}$.

R symmetry gives $\bar{L}_{0,\pm 1} - R_{0,\pm 1} = \{\mathbb{Q}, \cdot\}$ commuting with $L_{0,\pm 1}$.

2d and 4d central charges related by a negative factor.

3. Chiral algebra

Bootstrapping a W-algebra means solving for $\lambda_{12\mathcal{O}}$.

$$\mathcal{O}_1(z)\mathcal{O}_2(0) = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}} \sum_{m=0}^{\infty} \frac{(h+h_{12})_m}{m!(2h)_m} \frac{\partial^m \mathcal{O}(0)}{z^{h_1+h_2-h-m}}$$

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Crossing under $1 \leftrightarrow 3$ manifest but $1 \leftrightarrow 4$ must still be imposed.

$$F_{1234}(\chi) + \frac{(-1)^{k_1+k_4}\chi^{k_1+k_2}}{(\chi-1)^{k_2+k_3}} F_{3214}(1-\chi)$$

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All OPE coefficients fixed in Argyres-Douglas case.

$$F_{1234}^{I_1 I_2 I_3 I_4}(\chi;\beta) = \sqrt{\frac{6}{c_J}} f^{I_1 I_2 J} f^{J I_3 I_4} \sum_p g_{1-\frac{p}{2}}^{\frac{k_{21}}{2},\frac{k_{43}}{2}}(\beta^{-1}) \sum_{m=0}^{\frac{k_1+k_2-p-2}{2}} \frac{\left(\frac{p-k_{12}}{2}\right)_m \left(\frac{p+k_{34}}{2}\right)_m}{m!(p)_m \chi^{\frac{k_1+k_2-p}{2}-m}}$$

3. Chiral algbra and loops

Simplest chiral algebra correlator in 4d $\mathcal{N} = 4$ SYM.

$$F_{2222}(\chi;\alpha) = 1 + (\chi\alpha)^2 + \chi^2 \left(\frac{\alpha - 1}{1 - \chi}\right)^2 + \frac{12}{c}\left[\frac{\chi}{1-\chi} - 2\chi\alpha + \frac{(\chi\alpha)^2}{1-\chi}\right]$$

Why do we have $F_{2222}(\chi;\alpha)\big|_{1/c^2} = 0$ but $F_{3333}(\chi;\alpha)\big|_{1/c^2} \neq 0$?

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$$F_{3333}(\chi;\alpha)\big|_{1/c^2} = \frac{2025\chi^2\alpha(1-\alpha)}{c^2(1-\chi)}$$

Crossing continues to fix loops [CB, Ferrero, Zhou; 2101.04114] .

Unprotected data at one loop

For $[\mathcal{O}_2\mathcal{O}_2]_{n,\ell} \subset \langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2 \rangle$, it is standard to use $\mathcal{H}(\chi, \chi')$.

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$$\mathcal{H}(\chi, \chi') = \sum_{n,\ell} \left(a^{(0)}_{n,\ell} + a^{(1)}_{n,\ell} + \dots \right) G_{\Delta_{n,\ell} + \gamma^{(1)}_{n,\ell} + \dots, \ell}(\chi, \chi')$$

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Double-log is also a double-discontinuity defined by

$$dDisc[f(\chi,\chi')] = f(\chi,\chi') - \frac{1}{2}\left[f(\chi, e^{2\pi i}\chi') + f(\chi, e^{-2\pi i}\chi') \right].$$

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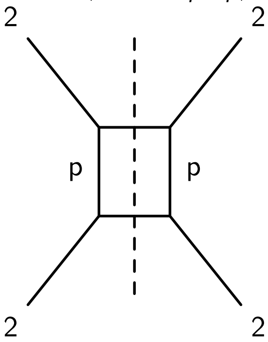
Full spectral density encoded in this [Caron-Huot; 1703.00278] !

$$c(\Delta,\ell) = \kappa_{\Delta,\ell} \int_0^1 \int_0^1 \left| \frac{\chi - \chi'}{\chi\chi'} \right|^{d-2} G_{\ell+d-1,\Delta+1-d}(\chi,\chi') dDisc[\mathcal{H}(\chi,\chi')] \frac{d\chi}{\chi^2} \frac{d\chi'}{\chi'^2}$$

Unprotected data at one loop

Use $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_p \mathcal{O}_p \rangle$ to get around $\langle \gamma^{(1)2} a^{(0)} \rangle_{n,\ell} \neq \frac{\langle \gamma^{(1)} a^{(0)} \rangle^2_{n,\ell}}{\langle a^{(0)} \rangle_{n,\ell}}$.



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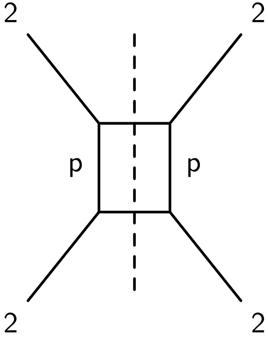

$$\begin{pmatrix} [\mathcal{O}_2\mathcal{O}_2]_1 \\ [\mathcal{O}_3\mathcal{O}_3]_0 \end{pmatrix} = \begin{pmatrix} \frac{\lambda_{22A}}{\sqrt{\lambda^2_{22A} + \lambda^2_{22B}}} & \frac{\lambda_{22B}}{\sqrt{\lambda^2_{22A} + \lambda^2_{22B}}} \\ \frac{\lambda_{33A}}{\sqrt{\lambda^2_{33A} + \lambda^2_{33B}}} & \frac{\lambda_{33B}}{\sqrt{\lambda^2_{33A} + \lambda^2_{33B}}} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

$$M = Q \begin{pmatrix} \gamma^{(1)}_A & 0 \\ 0 & \gamma^{(1)}_B \end{pmatrix} Q^T, \quad \langle \gamma^{(1)2} a^{(0)} \rangle_1 \in M^2$$

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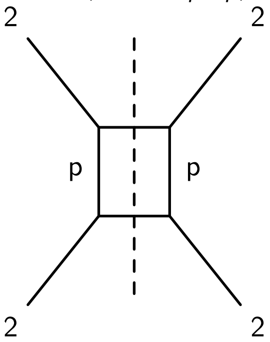

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Eigenvalues never involve higher roots [Aprile, Drummond, Heslop, Paul; 1706.02822] !

$$\gamma^{(1)}_{n,\ell,i} = -\frac{2(n+1)_4(n+\ell+2)_4}{(\ell+2i+1)_6}, \quad i = 0, \ldots, n$$

Hidden conformal symmetry

Higher dimensional blocks diagonalize this [Caron-Huot, Trinh; 1809.09173] .

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Graviton (gluon) amplitude conformal for $d = 10$ ($d = 8$)!

$$K_\mu = \sum_{i=1}^{4} \left[\frac{p_{i\mu}}{2} \frac{\partial}{\partial p_i} \cdot \frac{\partial}{\partial p_i} - p_i^\nu \frac{\partial}{\partial p_i^\nu} \frac{\partial}{\partial p_i^\mu} - \frac{d-2}{2} \frac{\partial}{\partial p_i^\mu} \right]$$

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Background $AdS_5 \times S^5$ and brane locus $AdS_5 \times S^3$ are both conformally flat. Corrections in λ also organize this way [Caron-Huot, Coronado; 2106.03892] .

An old conjecture

Actions for different d are perturbatively equivalent [Parisi, Sourlas; 1979] .

$$S = \int d^d x d\theta d\bar{\theta} \left[-\frac{1}{2} \Phi \partial^2 \Phi + V(\Phi) \right] \leftrightarrow S = \int d^{d-2} x \left[-\frac{1}{2} \phi \partial^2 \phi + V(\phi) \right]$$

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LHS invariant under $\mathfrak{osp}(d+1, 1|2)$ superalgebra.

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Superblock can be expressed in two ways [Kaviraj, Rychkov, Trevisani; 1912.01617] .

$$G^{(d-2)}_{\Delta,\ell} = G^{(d)}_{\Delta,\ell} + c_{2,0} G^{(d)}_{\Delta+2,\ell} + c_{1,-1} G^{(d)}_{\Delta+1,\ell-1} + c_{0,-2} G^{(d)}_{\Delta,\ell-2} + c_{2,-2} G^{(d)}_{\Delta+2,\ell-2}$$

Five term relation has two terms for $\ell = 0$.

Parisi-Sourlas SUSY in holography

Residues of $\mathcal{S}_p$ involve $K_p^{i,j}(t,u)H_{p,m}^{i,j}$ or $K_p^i(t,u)H_{p,m}^i$.

$K_p^{i,j}(t,u)$	$\hat{K}_p$
$t + \Delta_1 - \Delta_4 - 2\epsilon\mathcal{E}$	$2V\partial_V$
$u - \Delta_1 - \Delta_3$	$-2U\partial_U - 2V\partial_V$
i, j	$\sigma\partial_\sigma, \tau\partial_\tau$

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Special linear combinations are acted on by $\hat{K}_p$ [CB, Ferrero, Zhou; 2101.04114] .

$$\begin{aligned}
\mathcal{M}^{(d-2)}_{\epsilon p,0} &= \mathcal{M}^{(d)}_{\epsilon p,0} + c^{(d)}_{2,0}\mathcal{M}^{(d)}_{\epsilon p+2,0} \\
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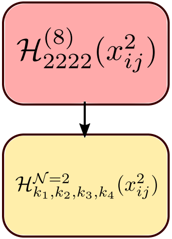
\end{aligned}$$

AdS_{d+1} and S^{n-1} dimensionality both reduce by same amount.

$$\mathcal{S}_p(x_i,t_i) = C(k_i,p)\widehat{K}_p \circ \left[W^{(d-\#Qs/4)}_{\epsilon p,0}(x_i)\,Y^{(n-\#Qs/4)}_p(t_i)\right]$$

Double copy is manifest for auxiliary AdS_5 correlators [Zhou; 2106.07651] .



More relations

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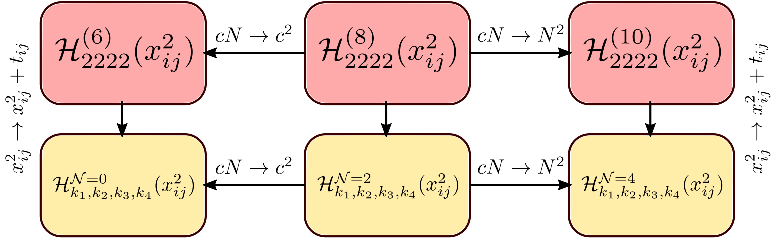

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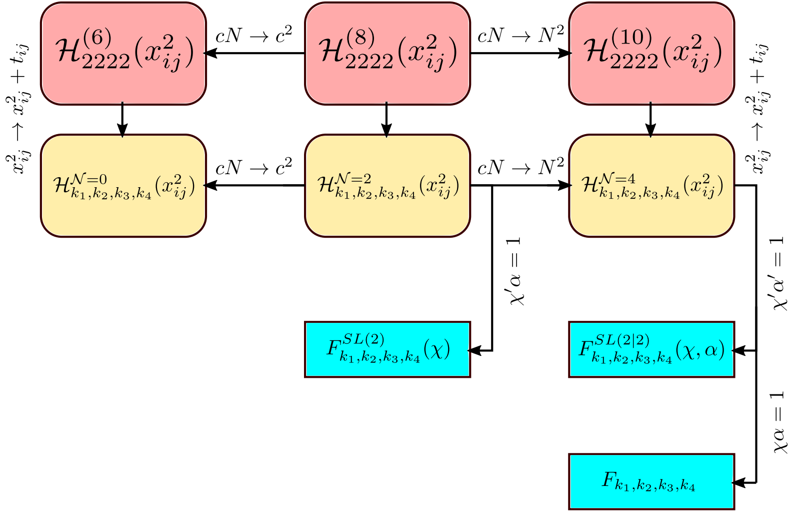

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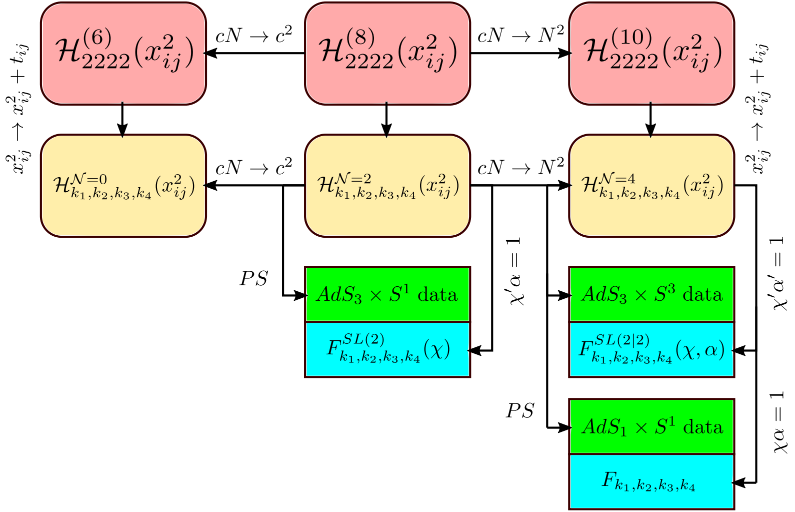

Conclusions

- Explicit Witten diagrams are being replaced by more elegant bootstrap methods.
- Some of the structures revealed by them still have a mysterious origin.
- The chiral algebra and AdS unitarity method both enable a systematic exploration of loops.
- Possible to consider both gravitons $O(1/c_T)$ and gluons $O(1/c_J)$ to study backreaction on the brane.
- Future targets include S-fold theories and backgrounds with defects.